Conference Program

Antalya Algebra Days VI

May 19–23, 2004
Most welcome to the sixth Antalya Algebra Days.

We can now say with confidence that Antalya Algebra Days has become a tradition. More than a tradition, this meeting of a few days is a need for most of us. We owe its existence to Cemal Koç, Mahmut Kuzucuoğlu, and especially Ayşe Berkman, an indispensable organizer.

This year the number of participants has increased by more than 50 percent. We also have many more talks than the previous years. Clearly we need a longer period of meeting in the future.

In exchange, the average age has decreased. For many of us, this is our first mathematical conference.

The number of our foreign visitors has been increasing. We thank them all for sharing their knowledge and ideas with us. We are sorry that our budget is too limited to support the travel and even the local expenses of our visitors. In exchange, we can offer abundant sea and sky and history and everything which is given for free to us, plus of course a cordial atmosphere.

As in the previous two meetings, Tamer Koç is helping us with the local organization. He is the essential worker and the backbone of the meeting. David Pierce is our webmaster and TeXnician. Esra Şengelen has been very efficient in coordinating and arranging the too-many little details. Ayşe Berkman has been at the other end of the internet whenever needed, and she has often been needed. Our main sponsor, as usual, is TÜBİTAK, the Scientific and Technical Research Council of Turkey. Additionally, this year, Istanbul Bilgi University and the Turkish Association of Mathematics supported the meeting financially. I would like to thank them all.

I hope that your visit to Antalya will be one of the most memorable and pleasant days of your life. Enjoy it!

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Talks and Speakers

New Aspects of the Sixty-Five-Year-Old Hopkins–Levitzki Theorem
   Toma Albu ....................................................... 8

δ-Supplemented Modules
   Mustafa Alkan .................................................. 9

$L_\alpha$ Modules and Induction from Elementary Abelian Subgroups
   Fatma Altunbulak ................................................. 9

Dunwoody Parameters $\gamma\alpha_0\beta_0\gamma_2$ and Abstract Groups $S((d+1)/2, d)$
   Nurullah Ankaralioğlu and Hüseyin Aydın ..................... 10

Stabilizers and Ascending Subgroups of Finitary Permutation Groups
   A.O. Asar .......................................................... 10

On the $n$-Centralizer Finite Groups, $n \leq 7$
   Ali Reza Ashrafi .................................................. 11

Stenitz Classes and Discriminant Counting
   Ebru Bekyel ....................................................... 12

Frobenius difference equations in Witt vectors
   Luc Bélair .......................................................... 13

Pseudofinite Fields and Random Graphs
   Özlem Beyarslan ................................................ 14

Some minimal but inefficient monoid presentations
   Ahmet Sinan Çevik .............................................. 14

Model theory of generic difference fields and some applications
   Zoé Chatzidakis ................................................... 15

Canonical Induction Formula for $K$-characters of a Finite Group
   Olcay Coşkun ..................................................... 15

On the distance between intervals in $C^\ast$-algebras
   Eduard Emel’yanov ............................................. 16

A remark on the algebra homomorphism
   Zafer Ercan ....................................................... 17

Introduction to Topological Algebra
   Ahmet Feyzioglu ................................................ 17

Cyclic Codes and Elementary Abelian Extensions
   Cem Güneri ....................................................... 18

The Functoriality Principle
   İlhan İkeda ....................................................... 18

A Characterization of $PSU(p,q)$ for some prime numbers $p$
   A. Iranmanesh ................................................... 19

Carter subgroups of groups of finite Morley rank
   Eric Jaligot ....................................................... 20
On Quadratic Poisson Brackets
Ali Ulas Ozgur Kisisel .................................................. 21

Several problems about $p$–groups
Sergey Gennadevich Kolesnikov ...................................... 22

Multiplicative properties of the dual (semi)-canonical basis
Bernard Leclerc ................................................................. 24

Symmetric forms and quadrics of projective spaces over local rings
V.M. Levchuk ................................................................. 25

The Abel-Jacobi Map for Higher Chow Groups
James D. Lewis ............................................................. 26

Galois Cohomology of Surgical Fields
Amador Martin-Pizarro .................................................... 27

A Constructive Improvement on the Tsfasman–Vlăduţ–Zink and Xing Bounds
Ferruh Özbudak .............................................................. 28

Dedekind Modules
M. Alkan, B. Saraç and Y. Tiraş ........................................ 29

Ultraproducts of commutative rings
A. Serpil Saydam .............................................................. 30

Vague Algebraic Structures and Their Fundamental Properties
Sevda Sezer ................................................................. 30

The Auslander conjecture for groups leaving a form of signature $(n – 2, 2)$ invariant
G. Soifer ................................................................. 31

On $l$-modular representations of finite groups of Lie type: Conjectures and recent results
Bhama Srinivasan .............................................................. 32

Automatic Semigroups
Rick Thomas ................................................................. 33

Asymptotic cones of uniform lattices
Simon Thomas ................................................................. 34

The Automorphism Tower Problem
Simon Thomas ................................................................. 34

Infinite-dimensional general linear groups are groups of universally finite width
Vladimir Tolstykh .......................................................... 35

Theoretical Analysis of Pseudorandom Number Sequences
Alev Topuzoğlu ............................................................ 36

An induction theorem for the unit groups of Burnside rings of 2-groups
Ergün Yalçın ................................................................. 36

Conjugately dense subgroups in free products of groups
Sergey Zyubin ............................................................... 36
Abstracts

New Aspects of the Sixty-Five-Year-Old
Hopkins–Levitzki Theorem

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The Classical Hopkins–Levitzki Theorem, discovered independently in 1939 by C. Hopkins and J. Levitzki states that any right Artinian ring with identity is right Noetherian, or equivalently, it can be reformulated as:

Classical H–L: Let $R$ be a right Artinian ring with identity. Then any Artinian right $R$-module is Noetherian.

In the last 30 years it has been generalized by Albu, Albu & Năstăscu, Albu & Smith, Miller & Teply, Năstăscu, etc., as follows:

Relative H–L: Let $R$ be a ring with identity, and let $\tau$ be a hereditary torsion theory on $\text{Mod}-R$. If $R$ is $\tau$-Artinian, then any $\tau$-Artinian right $R$-module is $\tau$-Noetherian.

Absolute H–L: Let $\mathcal{G}$ be a Grothendieck category having an Artinian generator. Then any Artinian object of $\mathcal{G}$ is Noetherian.

Latticial H–L: Let $L$ be an arbitrary modular Artinian lattice with $0$ and $1$. Then $L$ is Noetherian if and only if $L$ satisfies two conditions, one of which guarantees that $L$ has a good supply of essential elements and the second one ensures that there is a bound for the composition lengths of certain intervals of $L$.

The aim of this talk is to explain to a general audience all these aspects of the Classical Hopkins–Levitzki Theorem, their dual formulations, the connections between them, their applications to the investigation of the structure of some relevant classes of modules including injectives and projectives, as well as to present other newer aspects of it involving the concepts of Krull and dual Krull dimension.
\textbf{\(\delta\)-Supplemented Modules}

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In this work, the author gives a partial characterization of \(\delta\)-supplement submodules and projective \(\delta\)-covers. He provides conditions under which a given \(R\)-module \(M\) is \(\delta\)-semiregular iff every finitely generated submodules of \(M\) has a \(\delta\)-supplement. The bearing of this result upon the \(\delta\)-semiperfect rings are discussed.

\textbf{\(L_\zeta\) Modules and Induction from Elementary Abelian Subgroups}

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Let \(G\) be a finite group and \(k\) be a field of characteristic \(p\) where \(p\) is a prime number dividing the order of \(G\). In [1], Jon F.Carlson proves the following theorem

\textbf{Theorem.} There exists an integer \(\tau = \tau(G,p)\), depending only on \(G\) and \(p\) and finitely generated \(kG\)-module \(V\) such that the direct sum \(k \oplus V\) has a filtration \(\{0\} = L_0 \subseteq L_1 \subseteq ... \subseteq L_\tau = k \oplus V\) with the property that for each \(i = 1, 2, ..., \tau\) there is an elementary abelian \(p\)-subgroup \(E_i \subseteq G\) and a \(kE_i\)-module \(W_i\) such that \(L_i/L_{i-1} \cong W_i \uparrow^G\) where \(W_i \uparrow^G\) denotes the induced module \(W_i \uparrow^G = kG \otimes_{kE_i} W_i\).

In my master thesis I give an alternative proof to this theorem using \(L_\zeta\) modules. In this talk, I will give the definition of \(L_\zeta\) modules and explain some parts of this alternative proof.

\textbf{References}

Dunwoody Parameters \((1, b, 0, 2)\) and Abstract Groups \(S((d + 1)/2, d)\)

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We have shown in this paper that cyclically presented groups obtained using the word \(w\) connected with Dunwoody parameters \((a, b, c, r) = (1, b, 0, 2)\) are associated with the abstract groups \(S((d + 1)/2, d)\) when \(b\) is odd and \(d = 2a + b + c\). We also gave two different examples to illustrate how to apply our result to particular problems in application.

References


Stabilizers and Ascending Subgroups of Finitary Permutation Groups

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Let \(A\) be an infinite set and let \(\text{Sym}(A)\) be the symmetric group on \(A\). For each element \(x\) of \(\text{Sym}(A)\), the set of all the elements \(i\) of \(A\) such that \(x(i) \neq i\) is called the support of \(x\) and denoted by \(\text{supp}(x)\). If \(\text{supp}(x)\) is finite, then \(x\) is called a finitary permutation. The set of all the finitary permutations on \(A\) forms a subgroup of \(\text{Sym}(A)\), which is denoted by \(\text{FSym}(A)\). Let \(G\) be a subgroup of \(\text{FSym}(A)\) and \(a \in A\). Then the set \(Ga\) consisting of all the elements \(g\) of \(G\) such that \(g(a) = a\) is called the stabilizer of \(a\) in \(G\), and the set \(G(a)\) consisting of all \(g(a)\) as \(g\) ranges over \(G\) is called the orbit of \(G\) containing \(a\). A group \(G\) is called an FC-group if for each element \(a\) the set consisting of all \(g^{-1}ag\)
is finite. $G$ is called a **minimal non-FC-group** if $G$ is not an FC-group, but every proper subgroup of $G$ is an FC-group. Note also that $G$ is called **perfect** if it is equal to its commutator subgroup.

The purpose of this talk is to present some results on finitary permutation groups on an infinite set. Thus let $G$ be a transitive group of $\text{FSym}(A)$, where $A$ is an infinite set. We give some sufficient conditions under which the structure of $G$ can be determined from the structure of a point stabilizer which also enable us to generalize some of the earlier results on the subject. Also we give a characterization of ascending subgroups of $G$ and determine the conditions under which $G$ can be the product of two proper ascending subgroups. Finally we show that if $G$ is a perfect $p$-group such that every orbit of every proper subgroup of $G$ is finite, then it cannot be generated by a subset of finite exponent. We also give an example of a perfect totally imprimitive group in which the stabilizers satisfy the normalizer condition but the group is not a minimal non FC-group.

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### On the $n$-Centralizer Finite Groups, $n \leq 7$

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For a finite group $G$, $\# \text{Cent}(G)$ denotes the number of centralizers of its elements. A group $G$ is called $n$-centralizer if $\# \text{Cent}(G) = n$, and primitive $n$-centralizer if $\# \text{Cent}(G) = \# \text{Cent}(\frac{G}{\text{Z}(G)}) = n$.

In some research papers the problem is posed to find the structure of finite $n$-centralizer and primitive $n$-centralizer groups, for a given positive integer $n$. In this talk, we report the recent works on this problem.

### References


Stenitz Classes and Discriminant Counting

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Let $G$ be a finite abelian group and $k$ be a number field. Class field theory guarantees that there exists a number field $k'$ such that the Galois group of the extension $k'/k$ is isomorphic to $G$. A quantitative result about the density of such extensions $k'/k$ with Galois group isomorphic to $G$ is given by Wright [2] where counting is done using the norm of the discriminant of $k'/k$. The discriminant of an extension is classically defined as an ideal which carries information on the module structure of the ring of integers $\mathcal{O}$ and $\mathfrak{o}$ of the fields $k'$ and $k$, respectively. In particular if $\mathcal{O}$ is a free module over $\mathfrak{o}$ then the discriminant is a principal ideal. The converse of the statement requires the finer definition of the discriminant as an idéle. For a given ideal class $c$ in the ideal class group of $k$, the question of existence of an extension $k'/k$ with discriminant belonging to that class is settled by McCulloh in [1]. In this talk we will give a combination of the two results above by giving asymptotics of the counting function of normal extensions $k'/k$ with Galois group isomorphic to $G$, the discriminant ideal $(\Delta (k'/k))$ belonging to $c$ and norm of $(\Delta (k'/k))$ less than or equal to $X$. The method used in proof is applying class field theory to construct the corresponding Dirichlet series of the counting function and then applying a Tauberian theorem to translate the information about the poles to asymptotic behavior. We also prove that the existence of a single $k'/k$ with the desired properties is sufficient for a positive density result.

References


Frobenius difference equations in Witt vectors

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Let $A$ be either the ring $W[k]$ of Witt vectors over an algebraically closed field $k$ of characteristic $p > 0$, or the ring $k[[T]]$ of formal power series over a field $k$ of characteristic 0. Let $t \in A$ stand for either the prime number $p$, in case $A = W[k]$, or the indeterminate $T$, in case $A = k[[T]]$. The ring $A$ has characteristic 0, the non-invertible elements form an ideal $M$ which is generated by $t$, and the quotient ring $A/M$ is a field isomorphic to $k$. Let $\varphi$ be an automorphism of $k$; then there is a natural lifting $\tilde{\varphi}$ of $\varphi$ to an automorphism of $A$. In case $A = k[[T]]$, then $\tilde{\varphi}(\sum a_n T^n) = \sum \varphi(a_n) T^n$. The case of interest for $A = W[k]$ is $\varphi = x \mapsto x^p$, and then the lifting $\tilde{\varphi}$ can be viewed somewhat as above. We will require that $k$ and $\varphi$ satisfy the condition that any equation of the form $\varphi'(X) + c_{i-1} \varphi^{i-1}(X) + \cdots + c_1 \varphi(X) + c_0 X + c = 0$ has a solution in $k$, $\varphi^i$ being the $i$-th iterate of $\varphi$. Here, the Frobenius map satisfies this requirement. We will denote by $\sigma$ an automorphism of $A$ as above. Consider a system of equations

$$F_1 = 0, \ldots, F_m = 0,$$  

(*)&

where the $F_i$ are polynomials over $A$ in the iterates $\sigma^i(X_j)$ of $\sigma$ and variables $X_j$. In case of just one variable $X$, these are of the form $\sum a_{i,j} \sigma^i(X)^j$. With some extra assumptions if $A = k[[T]]$ we have

**Theorem.** If, for all integer $n \geq 1$, the system (*) has a solution mod $t^n$, then it has a solution in $A$.

**Corollary.** There exists an integer $n_0 \in \mathbb{N}$, which depends only on the number and degree of the $F_i$ and the number of variables $X_j$, such that if the system (*) has a solution mod $t^{n_0}$, then it has a solution in $A$.

We adapt A. Robinson’s proof of the well known similar result for algebraic equations. We discuss some consequences and more general results.

**References**


Pseudofinite Fields and Random Graphs

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A pseudofinite field is an infinite field satisfying all first-order properties in the language \{0, 1, +, \times\} of rings which hold in all finite fields, like having an extension of degree \(p\) for every prime \(p\). Pseudofinite fields exist and they can be realized for example as ultraproducts of finite fields.

An \(n\)-ary random graph is a set \(X\) with a symmetric and irreflexive \(n\)-ary relation \(R\) (i.e. \(R \subseteq X^n\), \(R\) is \(\text{Sym}(n)\)-closed and no two entries of an element of \(R\) are equal) such that for any two finite and disjoint subsets \(A\) and \(B\) of \(X^{n-1}\), there is an \(x \in X\) such that \(R(a, x)\) and \(\neg R(b, x)\) for all \(a \in A\) and \(b \in B\).

In 1976 J.L. Duret interpreted a random binary graph in a pseudofinite field. This has some important model theoretic consequences.

We will discuss about possible ways to interpret the random \(n\)-ary graph in pseudofinite fields.

Some minimal but inefficient monoid presentations

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In recent papers of Ruskuc (1996), Saito (1989) and J.Wang (1998), the semi-direct product of arbitrary two monoids and a standard presentation, say \(\mathcal{P}\), for this product have received considerable attention. Also Wang defined a trivialiser set of the Squier complex associated with \(\mathcal{P}\). After that the necessary and sufficient conditions for \(\mathcal{P}\) to be efficient have been given by Çevik (2003). In this talk I will give the sufficient conditions for a presentation of the semi-direct product of a one-relator monoid by an infinite cyclic monoid to be minimal but not efficient.
Model theory of generic difference fields and some applications

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A difference field is a field with a distinguished endomorphism $\sigma$. The first results in Difference algebra were obtained by Ritt in the 30’s, and parallel his results on Differential algebra. The model-theoretic study of difference fields was started in the 90’s by Macintyre, Van den Dries and Wood, and continued by Hrushovski, Peterzil and myself, Pillay, . . . .

A difference field $K$ is generic if it is an existentially closed difference field, i.e., every finite system of difference equations over $K$ which has a solution in a difference field extending $K$ has already a solution in $K$. Note that in a generic difference field, the endomorphism is necessarily onto. The class of generic difference fields is elementary, and its theory is decidable. While it does not eliminate quantifiers, it comes quite close to it (every definable set is the finite-to-one projection of a set defined by difference equations).

In this talk I will first recall some of the basic results obtained on the model theory of these fields, and in particular the characterisation of non-modular types. Then I will show how these results were used by Hrushovski in the Manin–Mumford conjecture:

**Theorem.** Let $G$ be a commutative algebraic group defined over a number field, and let $X$ be a subvariety of $G$. Then

$$X \cap \text{Tor}(G) = \bigcup_{i=1}^{M} a_i + \text{Tor}(G_i)$$

for some subgroups $G_i$ of $G$, elements $a_i$, and the number $M$ can be bounded effectively. Here $\text{Tor}(G)$ denotes the torsion subgroup of $G$.

The Manin–Mumford conjecture had been proved earlier, and in somewhat more generality (no restriction on the field of definition of $G$). Hrushovski’s proof is quite novel, and yields effectivity of the bound. If time allows, I will also mention the following result of Hrushovski (independently announced by Macintyre):

**Theorem.** Let $Q$ be the set of prime powers, and for each $q = p^n \in Q$, let $K_q$ be the difference field $\mathbb{F}_p$ with distinguished automorphism $x \mapsto x^q$. Let $\mathcal{U}$ be a non-principal ultrafilter on $Q$. Then $\prod_{q \in Q} K_q/\mathcal{U}$ is a generic difference field.

The proof of this result entails giving estimates of the number of solutions of certain sets of difference equations. These estimates are similar in nature to the Lang–Weil estimates on the number of rational points in varieties over finite fields, and allow Hrushovski to prove the Jacobi’s conjecture for difference equations (an analogue of the Jacobi’s conjecture for differential equations).
Canonical Induction Formula for K-characters of a Finite Group

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Brauer’s induction theorem states that each complex character of a finite group can be written as an integral linear combination of monomial characters [i.e characters induced from one-dimensional characters]. This theorem has several generalizations, two of which are to modular characters and to K-characters for a subfield K of complex numbers \(\mathbb{C}\), known as the Witt-Berman Theorem. Brauer’s theorem, as well as the generalizations, is an existence theorem and there are three independent constructions for a canonical induction formula, that is explicit formula which gives the coefficient of a monomial character for a given character, by V. Snaith [2], P. Symonds [3] and R. Boltje [1]. Among these Boltje’s construction has a generalization to a special class of Mackey functors, called \(k\)-Mackey functors for a commutative ring \(k\). In particular, we get canonical induction formulae for various representation rings, for example to ring of modular characters or to ring of linear source modules.

The Witt-Berman theorem says that the map

\[
\bigoplus_H \mathcal{R}_K(H) \rightarrow \mathcal{R}_K(G)
\]

where \(\mathcal{R}_K\) denote the ring of K-characters for a field \(K \leq \mathbb{C}\) and \(H\) runs over certain subgroups of \(G\) which are semidirect products of cyclic \(p^i\)-group over \(p\)-groups is surjective.

In this talk, we show that by making a suitable choice of \(k\)-restriction functor, Boltje’s canonical induction formula yields more generally an integral canonical induction formula for K-characters.

References


On the distance between intervals in $C^*$-algebras

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We investigate the behavior of the distance between intervals in $C^*$-algebras
and in preduals of von Neumann algebras.

A remark on the algebra homomorphism

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Let $X$ be a realcompact Hausdorff space and $C(X)$ be the algebra of realvalued
continuous functions on $X$. It is well known that each algebra homomorphism
from $C(X)$ into $R$ is one point evaluation. In this work we present a simple and
direct proof of this result without using of Axiom of choice.

Introduction to Topological Algebra

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First lesson: Basic theory of topological groups, interaction of algebraic and
topological aspects, subgroups and subspace topology, products and product
topology, factor groups and quotient topology.
Second lesson: Isomorphism theorems, topological rings and division rings.
Third lesson: Topological vector spaces, cartesian topologies, division rings over
which the dimension characterizes finite dimensional vector spaces.
Cyclic Codes and Elementary Abelian Extensions

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One of the standard problems in the study of cyclic codes is to obtain bounds on their minimum weights. Around late 80’s, it was realized that algebraic curves over finite fields can be used for the weight analysis of cyclic codes. Namely, via Hilbert’s Theorem 90, the weight of a codeword in a cyclic code over $\mathbb{F}_q$ is related to the number of $\mathbb{F}_q^m$-solutions to the equation of the form $F(x,y) = y^q - y - f(x) = 0$, where $q$ is a power of a prime $p$ and $f(x) \in \mathbb{F}_q^m[x]$ for some $m > 1$. Using this observation Wolfmann gave a bound on the minimum weight. However, as is the case with other well-known coding theoretic bounds, Wolfmann’s result is not valid for any cyclic code. It applies only to those for which the related polynomials $F(x,y)$ are irreducible over $\mathbb{F}_q^m(x)$. In this case, $F(x,y) = 0$ defines an elementary abelian $p$-extension of the rational function field $\mathbb{F}_q^m(x)$ for which the genus can be computed and the Hasse-Weil bound can be found.

Together with Özbudak, we are able to extend the result of Wolfmann, i.e. extend the classes of cyclic codes. This is made possible by an explicit factorization of certain reducible polynomials of the form $F(x,y)$. The irreducible factors are more complicated than the original polynomial but they define a family of elementary abelian extensions which could be handled.

We will describe the general method, recall the necessary information on elementary abelian extensions (genus computation and irreducibility criteria), and show the types of polynomials we can factor explicitly. The resulting bound on the minimum weight of cyclic codes will also be presented.

The Functoriality Principle

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A Characterization of $PSU(p, q)$ for some prime numbers $p$

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For an integer $n$, let $\pi(n)$ be the set of prime divisors of $n$. If $G$ is a finite group, then $\pi(G)$ is defined to be $\pi(|G|)$. The prime graph $\Gamma(G)$ of a group $G$ is a graph whose vertex set is $\pi(G)$, and two distinct primes $p$ and $q$ are linked by an edge if and only if $G$ contains an element of order $pq$. Let $\pi_i$, $i = 1, 2, \ldots, t(G)$ be the connected components of $\Gamma(G)$. For $|G|$ even, $\pi_1$ will be the connected component containing 2. Then $|G|$ can be expressed as a product of some positive integers $m_i$, $i = 1, 2, \ldots, t(G)$ with $\pi(m_i) = \pi_i$. The integers $m_i$'s are called the order components of $G$. The set of order components of $G$ will be denoted by $OC(G)$. If the order of $G$ is even, we will assume that $m_1$ is the even order component and $m_2, \ldots, m_{t(G)}$ will be the odd order components of $G$. The order components of non-abelian simple groups having at least three prime graph components are obtained by G. Y. Chen. The order components of non-abelian simple groups with two order components can be obtained.

In this paper, we prove that the simple groups $PSU(p, q)$, for some prime numbers $p$, are also uniquely determined by their order components. As corollaries of this result, the validity of a conjecture of J. G. Thompson and a conjecture of W. Shi and J. Bi both on $PSU(p, q)$ is obtained.
Carter subgroups of groups of finite Morley rank

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This is joint work with Olivier Frécon. The Morley rank is a notion which comes from model theory, and more precisely stability theory in model theory. It can be seen as an abstract notion of the Zariski dimension in algebraic geometry over an algebraically closed field. One of the main questions in stability theory is open since the late seventies and is known as the Cherlin-Zilber Conjecture. It postulates that an infinite simple group of finite Morley rank is an algebraic group over an algebraically closed field.

Most of the work done towards this Algebraicity Conjecture in the last ten years has been consisting of trying to use ideas from the classification of the finite simple groups. This has been done modulo inductive assumptions for the Algebraicity Conjecture, as in the "revisionist" approach to the classification of the finite simple groups.

One of the main features of this talk is that it is free of these inductive assumptions. I will explain why any group of finite Morley rank contains a Carter subgroup, i.e. a definable connected nilpotent subgroup of finite index in its normalizer. This can be seen as an abstract version of the existence of maximal tori in (simple, say) algebraic groups.

Our proof of the existence of Carter subgroups uses a notion of 0-unipotence recently developed by Jeff Burdges in the context of groups of finite Morley rank, which corresponds to the notion of unipotence in characteristic 0 for algebraic groups. (Contrarily to algebraic group theory, such a notion is not immediately available in our context.) We build our Carter subgroups from their least unipotent part to their most unipotent part.
On Quadratic Poisson Brackets

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A Poisson bracket is a skew symmetric, bilinear map taking pairs of regular functions on a manifold to regular functions, which satisfies the Jacobi identity and the Leibniz rule. The type of manifold and regularity can be taken in any category that differentiation makes sense. Poisson brackets originated from physics, and eventually it was understood that they play the role of the classical limits of commutators in the quantum setting.

The purpose of this talk is to explain a method of constructing a large class of Poisson brackets enjoying a homogeneity property (corresponding to being a “quadratic bracket” in the terminology of completely integrable systems). We employ an analog of the Miura transformation from the theory of completely integrable systems to associative algebras. At the moment we know of two advantages of this method: first of all large families of interesting brackets may be constructed at once. Some (actually most) of these brackets are not constructible using R-matrices. Second, there are certain completely integrable systems for which the conserved quantities were known, but a Poisson bracket under which these conserved quantities commute was missing. Our method can be used to construct brackets of this type associated to those systems. We also expect that these ideas may be helpful to understand better the geometry of the parameter space of all quadratic Poisson brackets.
Several problems about $p$–groups

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We consider several problems connected with $p$–groups such as rationality, $k$–reflectionality, regularity and describing of automorphisms.

§1. A finite group $G$ is called rational if each of its complex irreducible characters takes only a rational values, and $k$–reflectional if each of its elements is a product of $k$ or fewer involutions.

It well known that Weil groups are rational and $2$–reflectional [1],[2]. Alternating group $A_n$ is rational for all $n$ too, but it is $2$–reflectional if and only if $n = 5, 6, 10, 14$ [3]. We proved the following

**Theorem 1.** Sylow $2$–subgroups of Weil groups and alternating groups are rational and $2$–reflectional.

In particularly, its theorem answers on the following question raised by Ya. G. Berkovich in ([4], question 15.25): Is sylow $2$–subgroup of symmetric group $S_2$–rational?

§2. Let $\Phi$ be system of root of normal type. We denote by $S\Phi(Z_{p^m})$ sylow $p$-subgroup of Chevalley group type of $\Phi$ over ring $Z_{p^m}$ residue classes of integers modulo $p^m$ ($p$ – prime integer, $m \geq 1$ – integer). Automorphisms of $S\Phi(Z_{p^m})$ was studied in [5],[6],[7], when $m = 1$. We described its automorphisms for any $m \geq 1$. In particularly, for $p > 3$ is holds the following

**Theorem 2.** Any automorphism of group $S\Phi(Z_{p^m})$ for $p > 3$ and $m \geq 1$ is a product of standard — inner, diagonal, graph and central — automorphisms and also explicit determining special automorphisms of order $p$.

Obtained results answer the following question raised by V.M. Levchuk in ([4], question 12.42): To describe automorphisms of group $S\Phi(Z_{p^m})$ for any prime $p$ and any integer $m \geq 2$.

§3. A finite $p$–group $G$ is called regular if for any two its elements $a, b$ there exists such element $c$ from commutant subgroups $\langle a, b \rangle'$, that $(ab)^p = a^pb^pe^p$. In Kourovka Notebook B.A.F. Wehrfritz raised the following question ([4], question 8.3): For which $m$ and $n$ sylow $p$–subgroup $P_n(Z_{p^m})$ of general linear group $GL_n(Z_{p^m})$ is regular? In [8] answer was obtained for $m = 1$. We proved the following

**Theorem 3.** For $m = 1, 2$ group $P_n(Z_{p^m})$ is regular if and only if $n < (p + 1)/m$.

From theorem 3 following that groups $P_n(Z_{p^m})$ are not regular for any $m > 1$, when $n \geq (p + 1)/2$, since subgroups and factor groups of regular group are regular too. Thus, it remains research case $n \leq (p – 1)/2$ and $m > 1$. It seems in that case is hold the following

**Conjecture.** Groups $P_n(Z_{p^m})$ for $n \leq (p – 1)/2$ and any $m \geq 1$ are regular.
Validity of this conjecture indirectly confirm the following

**Theorem 4.** Denote by \( S^{(k)} \), \((k = 1, 2, \ldots)\) — members of low central series of group \( P_n(Z_{p^m}) \cap SL_n(Z_{p^m}) \). If \( p \geq 5 \) and \( n \leq (p - 1)/2 \) then \( [S^{(k)}]^p = S^{(k+n)} \), that is the set of \( p \)-powers elements from subgroup \( S^{(k)} \) form subgroup \( S^{(k+n)} \).

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Multiplicative properties of the dual (semi)-canonical basis

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Let \( \mathfrak{n} \) be a maximal nilpotent subalgebra of a simple complex Lie algebra \( \mathfrak{g} \), and let \( U(\mathfrak{n}) \) denote its enveloping algebra. In 1990 Lusztig and Kashiwara have introduced independently a canonical basis \( \mathcal{B} \) of \( U(\mathfrak{n}) \) with very interesting properties. For instance \( \mathcal{B} \) projects onto a basis of every finite-dimensional irreducible \( \mathfrak{g} \)-module.

Let \( G \) be a complex Lie group with Lie algebra \( \mathfrak{g} \) and \( N \) a maximal unipotent subgroup with Lie algebra \( \mathfrak{n} \). The algebra \( \mathbb{C}[N] \) of regular functions on \( N \) may be regarded as the dual of the Hopf algebra \( U(\mathfrak{n}) \). In 1993, Berenstein and Zelevinsky started the study of the dual canonical basis, that is, the basis \( \mathcal{B}^* \) of \( \mathbb{C}[N] \) which is dual to \( \mathcal{B} \). They showed that for \( \mathfrak{g} \) of type \( A_n \), \( (n \leq 4) \), each element of \( \mathcal{B}^* \) can be written in a unique way as a monomial in a finite set of elements of \( \mathcal{B}^* \), the set of “primes”. Moreover, the multiplicative structure of \( \mathcal{B}^* \) is controlled by a nice convex polytope – a generalized associahedra – whose faces correspond to these prime elements of \( \mathcal{B}^* \), and whose vertices encode the various types of allowed monomials.

Since then there has been several attempts to generalize these observations to other root systems. This is in particular one of the aims of the theory of cluster algebras recently developed in a series of remarkable papers by Fomin and Zelevinsky. But the situation is still very unclear, and for example there is yet no description of \( \mathcal{B}^* \) as above for \( \mathfrak{g} \) of type \( A_5 \).

In my talk I shall present some new joint results with Christof Geiss and Jan Schröer (math.RT/0402448) on these problems. Instead of \( \mathcal{B}^* \), we study the dual \( \mathcal{S}^* \) of the semicanonical basis \( \mathcal{S} \) of \( U(\mathfrak{n}) \), also introduced by Lusztig in the 90’s. We show that the multiplicative properties of \( \mathcal{S}^* \) reflect the geometric properties of the module varieties of a certain finite-dimensional algebra associated to \( \mathfrak{g} \), the preprojective algebra \( \Lambda \). In particular every element of \( \mathcal{S}^* \) has a unique factorization into a product of elements corresponding to indecomposable irreducible components of these varieties. For \( \mathfrak{g} \) of type \( A_n \), \( (n \leq 4) \), the algebra \( \Lambda \) has finite representation type and we can prove that \( \mathcal{S}^* = \mathcal{B}^* \). For type \( A_5 \), \( \Lambda \) is a tame algebra, and we give a completely explicit description of \( \mathcal{S}^* \) in terms of a 10-dimensional infinite root system of elliptic type \( E_8^{(1,1)} \).
Symmetric forms and quadrics of projective spaces over local rings

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Let $R$ be a local ring with $2 \in R^*$. We enumerate quadrics of a projective space $\mathbb{P}^{n-1}$, which is associated to a free module of rank $n$ over $R$. Every invertible symmetric matrix over $R$ is congruent to a diagonal matrix, by [3, §3]. However, examples show that the condition of invertibility is essential. Also, note that the Mobius, Minkowski and Laguerre’s classical geometries have natural representations by projective lines over algebras [1, Chapter 1]. The classical fundamental theorem of projective geometry over a field had been extended on the projective spaces over rings [4].

Consider the case of a local ring $R$ with a principal maximal ideal $J$ and $2 \in R^*$. It is proved that every symmetric matrix over $R$ is congruent to a diagonal matrix and the structure of the orthogonal group of any quadratic form over $R$ is described. Assume that the ideal $J$ is nilpotent of step $s$. If $[R^*: R^{*2}] = 1$, then the number of all classes of congruent $n \times n$ matrices (analogously, of equivalent matrices, see [2, 15]) is equal to $\binom{s+n}{n}$. Denote by $\binom{p}{q}$ the integer $p \geq q \geq 0$ and $0$ in other cases, by $\lfloor \alpha \rfloor$, the integral part of $\alpha$. Let $\Omega_q(m)$ be the set of all ordered collections $(n_1, \ldots, n_q)$ of integers $n_j > 0$ with sum $m$.

**Theorem 1.** Let $N$ be the number of all classes of projectively congruent quadrics of projective space $\mathbb{P}^{n-1}$ and let $[R^*: R^{*2}] = 2$. Then for cases $R^* \cap (1 + R^2) \not\subset R^{*2}$ and $1 + R^{*2} \subset R^{*2}$ the number $N$ is equal, respectively,

$$
\sum_{m=1}^{n} \min \{m, s\} \sum_{q=1}^{m} \binom{s}{q} 2^{q-1} \left\{ \binom{m/2 - 1}{q-1} + \binom{m-1}{q-1} \right\},
$$

$$
\sum_{m=1}^{n} \min \{m, s\} \sum_{q=1}^{m} \binom{s}{q} \left\{ \binom{m/2 - 1}{q-1} + \sum_{(n_1, \ldots, n_q) \in \Omega_q(m)} \frac{1}{2} \prod_{j=1}^{q} (n_j + 1) \right\}.
$$

This theorem is proved together with O.A. Starikova. The research is supported by Russian Fond of Basic Investigations, code of grant 03-01-00905.

**References**


The Abel-Jacobi Map for Higher Chow Groups

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This talk concerns various cycle class maps on certain higher algebraic cycle groups (introduced by S. Bloch) of a projective algebraic variety, having their values in certain computable cohomology theories. These cycle class maps are referred to as regulators, and can be thought of as a generalization of the logarithm.

For example, Dirichlet used the logarithm to define a map from the multiplicative group of a ring of algebraic integers to a real vector space. Then Dirichlet proved the celebrated analytic class number formula which relates all the important number theoretic invariants of the number field to the covolume of the Dirichlet regulator. Since the 1960’s Dirichlet’s fundamental discovery has been found potentially to occur elsewhere in number theory, in algebraic geometry, in class field theory, in algebraic K-theory, in the theory of algebraic cycles and motives, and in Hodge theory. Regulators come in many different forms, according to the context.

For instance, the Borel regulator is the higher-dimensional analogue of the Dirichlet regulator, considered as a map on algebraic K-theory in dimension one. On the other hand, in Riemann surface theory, the regulators might involve abelian integrals and Jacobians, extending the ideas of the 19th century analytic number theorists and geometers. Generally speaking, in its current incarnation, a regulator is a map from the algebraic K-theory of an algebraic variety to a suitable cohomology theory such as étale cohomology or Deligne cohomology.

In this talk, I will provide an explicit formula for the Bloch/Beilinson regulator for smooth projective varieties over the complex numbers. No knowledge of algebraic K-theory is required. A number of key “toy model” examples will be discussed, in keeping with the goal of making this talk accessible to a general audience.
Galois Cohomology of Surgical Fields

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Model theory studies definable sets within a given structure. It has become an important and active area of mathematics in the last decade. Its interactions with algebraic geometry are worth considering. Its applications to other areas of mathematics were underestimated until in 1996 Hrushovski gave a proof of the Mordell–Lang conjecture for function fields valid in all characteristics, using machinery from stability theory in a clever way. Similarly, he was able to derive a new proof of the Manin-Mumford conjecture for semi-abelian varieties over number fields. An active part of his results was the model-theoretical characterization of algebraically closed fields. They were shown to be \( \omega \)-stable.

The reason why such a characterization becomes important is because there is a notion of a rank in the above structures, that gives us a dimension on definable sets. In 1995 Pillay and Poizat considered which properties this dimension should satisfy. The key property is that, if a definable set \( U \) is mapped \textit{via} a finite-to-one map to another definable set \( V \), the dimension of \( U \) is smaller than the dimension of \( V \), and moreover, given a definable equivalence relation on a definable set, there are not infinitely many classes of maximal dimension. They called such structures \textit{surgical}. An important case was when there was a field definable in a surgical structure. Such a field is called a surgical field. Not all surgical structures arise from a rank, for example, \( \omega \)-minimal fields (for example, the real numbers) have the \( \omega \)-minimal dimension. In this very weak setting they showed that surgical fields were perfect and had small Galois group (\textit{i.e.} there are only finitely many non-isomorphic extensions of every finite degree).

In this talk, I will present an overview of the material and show that, given a surgical field \( K \) and a finite extension \( L \), the following holds: for every finite Galois extension \( L_1 \) of \( L \), the Brauer group \( Br(L_1/L) \) is finite. Moreover, given any algebraic group \( G \) defined over \( L \), we have that the first cohomology group of the extension \( L_1 : L \), denoted by \( H^1(L_1/L, G) \) is again finite. I will introduce what the aforementioned objects are and what we can conclude from their geometrical meaning.
A Constructive Improvement on the Tsfasman–Vläduţ–Zink and Xing Bounds

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Let \( \mathbb{F}_q \) be the finite field of order \( q \) and \( \alpha_q \) be the well-known function ([2], Section 1.3.1) in the theory of asymptotic algebraic codes. A central problem in algebraic coding theory is to find lower bounds on \( \alpha_q(\delta) \) for \( 0 < \delta < (q - 1)/q \).

A classical lower bound is the asymptotic Gilbert–Varshamov bound which says that

\[
\alpha_q(\delta) \geq 1 - \delta \log_q(q - 1) + \delta \log_q \delta + (1 - \delta) \log_q(1 - \delta) \quad \text{for} \quad 0 < \delta < \frac{q - 1}{q}.
\]

Let \( N_q(g) \) denote the maximum number of rational places that a global function field of genus \( g \) with full constant field \( \mathbb{F}_q \) can have. We recall the quantity

\[
A(q) = \limsup_{g \to \infty} \frac{N_q(g)}{g}
\]

from the theory of global function fields. In an important breakthrough, Tsfasman, Vläduţ, and Zink [3] showed that one can beat the asymptotic Gilbert–Varshamov bound by using Goppa’s algebraic-geometry codes [1]. The bound in [3] is

\[
\alpha_q(\delta) \geq 1 - \delta - \frac{1}{A(q)} \quad \text{for} \quad 0 \leq \delta \leq 1. \tag{\text{"}}
\]

Recently Xing [4] improved the Tsfasman–Vläduţ–Zink bound (\text{"}) and he showed that for any \( \delta \in (0, 1) \) we have

\[
\alpha_q(\delta) \geq 1 - \delta - \frac{1}{A(q)} + \sum_{i=2}^{\infty} \log_q \left( 1 + \frac{q - 1}{q^{2i}} \right). \tag{\text{"}}
\]

The proof of (\text{"}) given in [4] proceeds in a nonconstructive manner.

In this talk we present an improvement on the Xing bound (\text{"}) and thus also on the Tsfasman–Vläduţ–Zink bound (\text{"}). Moreover, the proof of our bound is obtained constructively in a certain range for \( \delta \). Namely we prove constructively that

\[
\alpha_q(\delta) \geq 1 - \delta - \frac{1}{A(q)} + \log_q \left( 1 + \frac{1}{q^3} \right) \tag{\text{"}}
\]

for any \( \delta \in \left[ 0, 1 - \frac{2}{A(q)} \right] - \left[ \frac{4q - 2}{(q - 1)(q^3 + 1)} \right] \).

This is a report on a joint work with Harald Niederreiter.
References


Dedekind Modules

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In this work, the authors introduce the concept of integral property for modules and give some characterizations for Dedekind modules and Dedekind domains with this property. They also show that a given ring $R$ is integrally closed if and only if a finitely generated torsion-free projective module over domain $R$ is integral. In addition, it is proved that any invertible submodule of a finitely generated projective module over a domain is finitely generated and projective. Also they give the equivalent conditions for Dedekind modules and Dedekind domains.
Ultraproducts of commutative rings

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This is joint work with Bruce Olberding, New Mexico State University. The focus of this talk is the structure of ultraproducts of commutative rings, and in particular, ultraproducts of integral domains. Ultraproducts of certain classes of integral domains, such as orders in algebraic number fields, have been well-studied but often from a model-theoretic point of view. Our goal in this talk is to give some new examples of how one can construct unusual rings using ultraproducts, then focus on the ring-theoretic properties of the ultraproducts of integral domains. The work contains some simple observations regarding first order properties of commutative integral domains. We analyze the $n$-generator property and coherency and use this analysis to construct a GCD domain that is not coherent, the existence of which was an open question until recently. Later, we turn to some of the main structural results of the work. These focus, for example, on the prime spectrum of an ultraproduct of integral domains and the representation of an ultraproduct of domains as an intersection of its localizations.

Vague Algebraic Structures and Their Fundamental Properties

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The study of fuzzy algebraic structures was started with the introduction of the concept of fuzzy subgroups by Rosenfeld in [5]. In Rosenfeld's paper, only the subsets were fuzzy, but the group operation was crisp. To get more general extension, Demirci [2, 3, 4] proposed the concept of “vague group” based on fuzzy equalities and fuzzy functions given in [1].

In this work, using the definition of vague group and its some results, we define some of vague algebraic concepts; for example, generalized vague subgroup, vague normal subgroup, vague subring, vague ideal, etc. We obtain fundamental properties of these concepts.

References

The Auslander conjecture for groups leaving a form of signature \((n - 2, 2)\) invariant

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This work is joint with H. Abels and G. Margulis. Let \(\text{Aff} \mathbb{R}^n\) be the group of affine transformations of the real affine space \(\mathbb{R}^n\). A subgroup \(\Gamma\) of \(\text{Aff} \mathbb{R}^n\) is called properly discontinuous if \(\{\gamma \in \Gamma; \gamma K \cap K \neq \emptyset\}\) is finite for every compact subset \(K\) of \(\mathbb{R}^n\). And \(\Gamma\) is called crystallographic if \(\Gamma\) is properly discontinuous and the orbit space \(\Gamma \backslash \mathbb{R}^n\) is compact. A subgroup \(\Gamma\) of \(\text{Aff} \mathbb{R}^n\) will also be called an affine group. A long standing conjecture of Auslander states that every affine crystallographic group \(\Gamma\) is virtually solvable. So far only special cases of this conjecture have been proved. The main result of this announcement deals with the following situation. Since \(\text{Aff} \mathbb{R}^n = GL(n, \mathbb{R}) \ltimes \mathbb{R}^n\) there is a natural homomorphism \(\ell: \text{Aff} \mathbb{R}^n \to GL(n, \mathbb{R})\), called the linear part. Let \(B\) be a non degenerate quadratic form on \(\mathbb{R}^n\) of signature \((p, q)\) and let \(O(B)\) be the orthogonal group of the form \(B\).

**Theorem.** Let \(\Gamma\) be an affine crystallographic group with \(\ell(\Gamma) \subset O(B)\). Then \(\Gamma\) is virtually solvable.

In the case under consideration this result settles the Auslander conjecture completely. To put this result into perspective let us recall the following results. Let \(\Gamma\) be an affine crystallographic group and suppose \(\ell(\Gamma) \subset O(B)\) for a non degenerate quadratic form \(B\) of signature \((p, q)\). Then \(\Gamma\) is virtually abelian if \(B\) is positive definite, i.e., \(q = 0\). This is an old theorem of Bieberbach. \(\Gamma\) is virtually solvable if \(q = 1\) (Goldmann, Kamishima). The content of the Theorem is that \(\Gamma\) is virtually solvable if \(q = 2\). The methods of our proof are completely different from the ones used for the case \(q \leq 1\).
On $l$-modular representations of finite groups of Lie type: Conjectures and recent results

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The theory of modular representations of a finite group was developed by Richard Brauer starting in the 1930’s. The classical theory of finite group representations studies the homomorphisms of an abstract finite group $G$ into a group of matrices over a field of characteristic 0, usually the field of complex numbers. By taking traces of the matrices, each such representation gives rise to a character, i.e. a function from the group into the field. The irreducible characters over $\mathbb{C}$ (called ordinary characters) form an orthonormal basis of the space of complex-valued class functions on the group, and yield a lot of information about the group. The number of ordinary characters is the number of conjugacy classes of the group, and this leads to a “character table” for the group.

In the modular theory one studies homomorphisms of the group $G$ into a group of matrices over a field $k$ of prime characteristic, usually a prime $p$ which divides the order of the group. In this case also Brauer defined characters of the irreducible representations as complex-valued functions on the group, and they are called Brauer characters. A good introduction to the theory is found in [Curtis and Reiner, Representation Theory of Finite Groups and Associative Algebras (Wiley, 1962), Ch.12].

Brauer then divided the ordinary characters into subsets called “blocks”, which reflect the decomposition of the group algebra of $G$ over the field $k$ into a sum of indecomposable two-sided ideals. He also defined the “decomposition matrix” $D$, the transition matrix between ordinary characters and Brauer characters. Some of the main problems in the modular theory are:

- Find the $p$-blocks of $G$
- Find the matrix $D$
- Find the values of the Brauer characters, e.g. the degrees (values at 1)
- Obtain “local-to-global information”, i.e. information such as the number of characters in a block of $G$, from similar information in a block of a “local subgroup”, i.e. the normalizer $N_G(P)$ of a $p$-subgroup $P$ of $G$.

We now specialize our group $G$ to be a finite group of Lie type, e.g. a finite general linear group or classical group. We can also take $G$ to be a symmetric group, in which case the $p$-blocks are known for every prime $p$, but the matrices $D$ are not known. If $G$ is $GL(n,q)$, $Sp(2n,q)$, $O(2n+1,q)$, $O^+(2n,q)$, $O^-(2n,q)$, we must look at two separate theories: whether the prime divides or does not divide $q$. From now on we assume that we are considering $l$-modular theory, where $l$ does not divide $q$. Then again the $l$-blocks are known, but the matrices
are not known. As for “local-to-global information”, there have been several conjectures put forward during the last few years. These are the Alperin-McKay conjectures, Dade conjectures, Isaacs-Navarro conjectures and Broué’s conjectures. These conjectures relate the characters in a block of $G$ with the characters in blocks of “local subgroups” of $G$. In this talk we will explain these conjectures and recent results concerning them in joint work with Paul Fong and Michel Broué.

**Automatic Semigroups**

Rick Thomas

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There have been some intriguing interactions in recent years between group theory and theoretical computer science. One area which has proved to be very fruitful in providing interesting and useful results is that of automatic groups where computation in the group can (essentially) be performed via an associated set of finite automata. For example, it is known that any automatic group must be finitely presented and the word problem of an automatic group can be solved in quadratic time. For an overview of automatic groups, see [1] and [3].

The purpose of this talk will be to introduce the notion of an automatic group, give some examples and mention some results. Our main motivations are to indicate how the definition can be extended to semigroups and to give an outline of some recent work establishing a theory of automatic semigroups.

We will aim to introduce the area and survey some of what is known (as opposed to giving proofs of results). We find that some of the results in automatic groups do generalize to semigroups (such as the solution of the word problem in quadratic time) whereas others do not (for example, there exist automatic semigroups that are not finitely presented).

For an introduction to automatic semigroups, see [2].

**References**


Asymptotic cones of uniform lattices

Simon Thomas

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If an observer moves steadily away from the Cayley graph of a finitely generated group, then any finite configuration will eventually become indistinguishable from a single point; but he may observe certain finite configurations which resemble earlier configurations.

The asymptotic cone is a topological space which encodes all of these recurring finite configurations. Unfortunately the construction of an asymptotic cone involves a number of non-canonical choices, and it was not clear whether the resulting asymptotic cone was independent of these choices. In this talk, partially answering a question of Gromov, I shall consider the question of whether uniform lattices in $SL(n, \mathbb{R})$ have unique asymptotic cones up to homeomorphism. The answer turns out to be very surprising! This is joint work with Linus Kramer, Saharon Shelah and Katrin Tent.

The Automorphism Tower Problem

Simon Thomas

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If $G$ is a centreless group, then there is a natural embedding of $G$ into its automorphism group $\text{Aut}(G)$, obtained by sending each $g \in G$ to the corresponding inner automorphism $i_g \in \text{Aut}(G)$. It is easily shown that $\text{Aut}(G)$ is also a centreless group. Hence we can define the automorphism tower of $G$ to be the ascending chain of centreless groups

$$G = G_0 \leq G_1 \leq G_2 \leq \ldots G_\alpha \leq G_{\alpha+1} \leq \ldots$$

such that for each ordinal $\alpha$

(a) $G_{\alpha+1} = \text{Aut}(G_\alpha)$; and

(b) if $\alpha$ is a limit ordinal, then $G_\alpha = \bigcup_{\beta < \alpha} G_\beta$.

(At each successor step, we identify $G_\alpha$ with the group $\text{Inn}(G_\alpha)$ of its inner automorphisms via the natural embedding.) The automorphism tower is said to terminate if there exists an ordinal $\alpha$ such that $G_{\alpha+1} = G_\alpha$. In 1939, Wielandt proved that if $G$ is a finite centreless group, then its automorphism tower terminates after finitely many steps. However, there exist natural examples of infinite centreless groups whose automorphism towers do not terminate after finitely many steps.

In this mini-course, we shall study automorphism towers of infinite centreless groups. In particular, we shall prove that the automorphism tower of an arbitrary infinite centreless group $G$ eventually terminates, perhaps after a transfinite number of steps; and also that for every ordinal $\alpha$, there exists an infinite
centreless group $G$ whose tower terminates after exactly $\alpha$ steps. We shall also discuss some of the many remaining open problems.

**Infinite-dimensional general linear groups are groups of universally finite width**

Vladimir Tolstykh

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Last summer George Bergman [1] proved that the symmetric group on an infinite set possesses the following property which we call the *universality of finite width*: given any generating set $X$ of the symmetric group on an infinite set $\Omega$, there is a uniform bound $k \in \mathbb{N}$ such that any permutation from $\text{Sym}(\Omega)$ is a product of at most $k$ elements of $X \cup X^{-1}$, or, in other words, $\text{Sym}(\Omega) = (X \pm 1)^k$.

In [1] Bergman also expressed his belief that further examples of groups of universally finite width might be found among ‘the automorphism groups of structures that can be put together out of many isomorphic copies of themselves’, and particularly mentioned, in this respect, infinite-dimensional linear groups.

Soon afterwards, strong supporting evidence for Bergman’s conjecture has been provided by several authors. For instance, the automorphisms groups of doubly homogeneous ordered sets [3] and the groups of autohomeomorphism of certain topological spaces [2] (such as, for instance, the Cantor discontinuum, $\mathbb{Q}$ and the set of irrational numbers) are groups of universally finite width.

**Theorem 1.** Let $V$ be an infinite-dimensional vector space over a division ring. Then the general linear group $\text{GL}(V)$ of $V$ is a group of universally finite width.

The proof is based on the ideas developed by Macpherson in [4] for the study of cofinalities of infinite-dimensional general linear groups and on the following fact.

**Proposition 2.** There is an involution $\pi$ from $\text{GL}(V)$ such that $\text{GL}(V)$ has finite width relatively to the conjugacy class $C = C(\pi)$ of $\pi$ (that is, $\text{GL}(V) = C^k$ for some natural $k$.)

**References**


Theoretical Analysis of Pseudorandom Number Sequences

Alev Topuzoğlu

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Nonlinear methods of generating pseudorandom numbers have many favorable properties when compared with linear congruential generators. Theoretical tests used in measuring the “randomness” of such sequences will be described and an overview of results on the structural properties, period length and distribution of generated points will be presented.

On the other hand nonlinear generators of higher orders are of particular interest as their period length can be made larger. Recent developments on the analysis of higher order nonlinear generators will also be discussed.

An induction theorem for the unit groups of Burnside rings of 2-groups

Ergün Yalçın

Bilkent University
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The talk will be a presentation of the following result: Let $B(G)^*$ denote the group of units of the Burnside ring of a 2-group $G$. We show that generalized induction map from the direct product of the unit groups $B(H/K)^*$ to $B(G)^*$ is surjective, where $H/K$ runs over the sections that are cyclic of order 2 or dihedral of order at least 16. As an application, we show that tom Dieck’s exponential map from the real representation ring to $B(G)^*$ is surjective. We also give a sufficient condition for the surjectivity of the exponential map from $B(G)$ to $B(G)^*$.

Conjugately dense subgroups in free products of groups

Sergey Zyubin

Tomsk Polytechnic University
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A subgroup of any group is called conjugately dense if it has nonempty intersection with each class of conjugate elements of the group. In the group $GL_n(K)$ over algebraically closed field, examples of proper conjugately dense subgroups are triangular subgroup, all it’s conjugate, and overgroups of these subgroups. In general, Borel subgroups and also parabolic subgroups are conjugately dense in any connected affine algebraic group. P. Neumann set the following problem.
in “Kourovka Notebook”. Describe all irreducible conjugately dense subgroups \( H \) of the group \( GL_n(K) \) over arbitrary field \( K \). In the same place P. Neumann conjectured that \( H = GL_n(K) \) with the exception of a case when \( n = \text{char} K = 2, K \) is quadratically closed field and \( H \) is conjugated to monomial subgroup [1, Problem 6.38a]. Finally he asked the following question. As far as it true that every conjugately dense subgroup of group of Lie type is parabolic? [1, Problem 6.38b] The author and V.M. Levchuk confirmed Neumann’s conjecture for the group \( GL_2(K) \) over a locally finite field \( K \) [2] and for the locally finite Chevalley groups of Lie rank 1 [3]. Further, the author proved that the conjecture is true for the group \( GL_3(K) \) over a locally finite field \( K \) [4]. In this paper we establish the following.

**Theorem 1.** Suppose the group \( G \) is a nontrivial free product \( G = G_1 *_{A} G_2 \) with amalgamated subgroup \( A, B \) be a largest normal subgroup of \( G \) that lying in \( A \), and quotient group \( G/B \) contain \( \alpha \) conjugacy classes. Suppose some group \( G_i, i = 1, 2 \) satisfies the next conditions: (1) \( |G_i : A| > 2 \); (2) \( \max\{|G_i : A|, n_0\} \geq \alpha \); (3) if \( g \in (G_1 \cup G_2) \setminus A \) and \( g^n \in A \) (\( n \in \mathbb{N} \)) then normal closure in the group \( G_i \) of subgroup, generated by element \( g^n \), lies in \( A \); then \( G \) have at least \( \max\{|G_i : A|, 2^n\} \) distinct conjugately dense subgroups.

Then we use Ihara’s decomposition of the group \( SL_2(K) \) over field \( K \) with discrete valuation into free product with amalgamation and prove the next.

**Theorem 2.** Let \( K \) be a field with nontrivial, discrete valuation and \( k \) be it’s residue class field. Suppose \( |K| = \max\{|k|, n_0\} \) then the group \( SL_2(K) \) has \( 2^{|K|} \) mutually nonconjugated conjugately dense subgroups.

The group \( SL_2(K) \) has only two conjugacy classes of parabolic subgroups: the group itself and Borel subgroups. It follows that Neumann’s conjecture is false for the group \( SL_2(K) \) over a field \( K \) that satisfies conditions of theorem 2. An example of such field is field of rational numbers with any \( p \)-adic valuation. Also we have another consequences of Theorem 1.

**Corollary 1.** Modular group \( PSL_2(\mathbb{Z}) \) has continuum mutually nonconjugated conjugately dense subgroups.

**Corollary 2.** Free group \( F(X) \) with rank \( > 1 \) has \( 2^{\max\{|X|, n_0\}} \) mutually nonconjugated conjugately dense subgroups.

This research was supported by the Russian Foundation for Basic Research (Grant 03–01–00905).

**References**


## Schedule

<table>
<thead>
<tr>
<th>Time</th>
<th>Wed 19th</th>
<th>Thurs 20th</th>
<th>Fri 21st</th>
<th>Sat 22nd</th>
<th>Sun 23rd</th>
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<tr>
<td>9:00</td>
<td>J.D. Lewis</td>
<td>S. Thomas</td>
<td>R. Thomas</td>
<td>S.G. Kolesnikov</td>
<td>E. Emel’yanov</td>
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<td>10:00</td>
<td>L. Bélaire</td>
<td>B. Srinivasan</td>
<td>A.O. Asar</td>
<td>E. Jaligot</td>
<td>V. Tolstykh</td>
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<td>coffee</td>
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<td>Z. Chatzidakis</td>
<td>O. Beyarslan</td>
<td>G. Soifer</td>
<td>A. Martin-Pizarro</td>
<td>parallel sessions</td>
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<td>12:30</td>
<td>lunch</td>
<td>lunch</td>
<td>lunch, recreation</td>
<td>lunch</td>
<td>lunch, farewell till next year!</td>
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<td>S.A. Zyubin</td>
<td>A. Topuzoğlu</td>
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<td>B. Leclerc</td>
<td>T. Albu</td>
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### Parallel sessions

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<td>Cem Gümêri</td>
<td>A. R. Ashrafi</td>
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<tr>
<td>17:00</td>
<td>Ebru Bekyel</td>
<td>Ali Özgür Kişisel</td>
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<td>Saturday</td>
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<tr>
<td>14:00</td>
<td>Ozlem Beyarslan</td>
<td>Vladimir Levchuk</td>
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<td>Serpil Saydam</td>
<td>Zafer Ercan</td>
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<td>B. Saraçoğlu</td>
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<td>Olcay Coşkun</td>
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<tr>
<td>11:30</td>
<td>Ali Iranmanesh</td>
<td>Ferruh Ozbudak</td>
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<tr>
<td>12:00</td>
<td>Ergün Yakın</td>
<td>Nurullah Ankarahöglü</td>
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