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Contents

I Invited talks

Mann pairs

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Cotorsion theories and pure-injectivity

Pedro A. Guil Asensio

Cotorsion theories in categories of modules have a special interest because of its close connection with Homotopical Algebra. In particular, it has been shown by Hovey [7] that they provide a very convenient framework for constructing Model Structures in the sense of Quillen. The aim of this talk is to study the structure of these cotorsion theories by means of a generalized notion of purity. As an application, we will construct a Monoidal Model Structure for additive categories embedded in Abelian Categories which allows to study the K-theory for these categories introduced by Quillen in [8].

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Model Theory of Mann Pairs

Ayhan Günaydın

(This is joint work with Lou van den Dries.) Let Ω be an ambient algebraically closed field, $\mathbf{k}$ a subfield, and G a subgroup of $\Omega^\times$. We say that $(\mathbf{k}, G)$ is a Mann pair if for every $n > 0$, there is a finite subset E of G^n such that for every $a_1, \dots, a_n \in \mathbf{k}$, and $g_1, \dots, g_n \in G$, if $a_1g_1 + \cdots + a_ng_n = 1$ and $\sum_{i \in I} a_ig_i \neq 0$ for all nonempty $I \subseteq \{1, \dots, n\}$, then $(g_1, \dots, g_n) \in E$. In this talk we show that if $(\mathbf{k}, G)$ is a Mann pair, then the elementary theory of the structure $(\Omega, \mathbf{k}, G)$ (the algebraically closed field Ω with distinguished subsets $\mathbf{k}$ and G) is determined by the elementary theories of $\mathbf{k}$ and G, after adding names for enough elements of $\mathbf{k}$ and G to witness that $(\mathbf{k}, G)$ is a Mann pair. Also under the additional assumption that $\mathbf{k}$ is algebraically closed we show that $(\Omega, \mathbf{k}, G)$ is stable.

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Blurred Complex Exponentiation

Jonathan Kirby

Boris Zilber constructed an exponential field, his "pseudo-exponential field", and conjectured that it is isomorphic to the complex exponential field. This conjecture is very hard, as it includes Schanuel's conjecture of transcendental number theory. I will explain how to *blur* an exponential field in a way which preserves the natural closure operator. Then I will give an idea of the proof that the results of blurring the complex exponential field and of blurring the pseudo-exponential field are isomorphic.

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Iterative differential equations

Piotr Kowalski

A theorem of Ax gives Schanuel-like conditions on the set of solutions of the differential equation of the exponential map. I will discuss possible generalizations of Ax's theorem to the case of positive characteristic. One needs to replace the exponential with an appropriate formal map and a derivation with a Hasse-Schmidt derivation.

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Residues of Algebraic Cycles

James D. Lewis

One of the deepest conjectures in the subject of algebraic cycles is the celebrated Hodge conjecture. This can be regarded as a statement on the Betti cycle class map for classical algebraic cycles. In the world of motives, there are the higher algebraic cycles introduced by S. Bloch. What is the appropriate statement of the Hodge conjecture in this context? In this talk we will give a user-friendly presentation of an amended version of Beilinson's formulation of the Hodge conjecture in terms of residues, and show how it relates to other conjectures due to Jannsen, Voisin and Bloch-Kato.

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Actions of weak Hopf algebras

Christian Lomp

1. Motivation. Group actions and group gradings as well as actions of envelopping algebras of Lie algebras are examples of Hopf algebra actions. Weak Hopf algebra (also called quantum groupoids) arose around 1998 in particle physics and were introduced by Böhm, Nil and Szlachányi in [1]. Examples of weak Hopf algebra actions are given by groupoid actions and certain von Neumann algebras. Here we will discuss a ring theoretical problem on the smash product $A\#H$ formed by a weak Hopf algebra H acting on an algebra A. In particular we will focus on the problem, when the smash product is semiprime or von Neumann regular.

2. Group actions, Group gradings and semisimple Hopf actions. If G is a finite group acting on an algebra A, then Bergman and Isaacs proved that if A has no $|G|$-torsion and is G-semiprime then A has a large fix ring, i.e. $I \cap A^G \neq 0$ for all G-stable ideals of A (see [2]). Moreover Fisher and

Montgomery proved in [3] that if A is semiprime, then the skew group ring $A * G$ is semiprime.

If A is G-graded, then Cohen and Montgomery showed in [4] that the smash product $A\#k[G]^*$ is semiprime provided A does not contain any non-zero nilpotent G-stable ideals.

In both cases A admits a Hopf action by $H = k[G]$ resp. $H = k[G]^*$ and Cohen and Fishman asked in [5] the natural question whether $A\#H$ is semiprime provided A is semiprime and H is semisimple.

Some positive answers can be given in terms of the separability of the smash product, and under additional finiteness conditions on A. If A is assumed to be commutative and reduced then $A\#H$ is semiprime by [6]. Recently Linchenko and Montgomery extended this result to semiprime algebras A which satisfy a polynomial identity [7].

4. Weak Hopf algebras. A weak Hopf algebra is roughly speaking a Hopf algebra whose counit and comultiplication are not necessarily unital maps. Proper examples of weak Hopf algebras are given by groupoid algebras, Hopf actions on separable algebras and certain Von Neumann algebras. We extend Linchenko and Montgomery's result to actions of a class of semisimple weak Hopf algebras on semiprime P.I. algebras .

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Algebraic properties of Zilber's exponential

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Hasse jets and prolongations

Rahim Moosa

The main purpose of this joint work with Tom Scanlon is to extend the Pillay-Ziegler construction of differential jet spaces in two ways: (1) from the finite-rank to possibly infinite-rank situation, and (2) from differential fields to a very abstract setting that includes Hasse-differential, difference, and difference-differential fields. To this end, I will introduce *generalised Hasse-rings* and develop an appropriate notion of prolongations. This will give rise to *generalised Hasse-varieties*. I will then introduce jet spaces for generalised Hasse-varieties, and show that in the separable case they determine the varieties.

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Arithmetic in groups of piece-wise affine permutations of an interval

Alexey Muranov

Valery Bardakov and Vladimir Tolstykh [1] showed recently that Richard Thompson's group F interprets the Arithmetic. In other words, F interprets the structure $(\mathbb{N}, +, \times)$ by first-order formulae with parameters. In this work we generalize their result in two directions. On the one hand, we use their approach to interpret $(\mathbb{N}, +, \times)$ in all the groups defined by Melanie Stein in [4], which include all the three groups of Thompson and all the finitely presented simple groups studied by Graham Higman in [3]. On the other hand, we show that the group F and some of its generalizations interpret $(\mathbb{N}, +, \times)$ *without parameters*.

Observe that if a structure of finite signature interprets the Arithmetic, then the elementary theory of this structure is *hereditarily undecidable*. (A theory of finite signature is called *hereditarily undecidable* if every its subtheory of the same signature is undecidable, cf. [5, Part I, §3].) Indeed, it is well known that $\mathrm{Th}(\mathbb{N}, +, \times)$ is hereditarily undecidable, cf. [5]. It is also known that if a structure N of finite signature interprets with parameters another structure M of finite signature such that $\mathrm{Th}(M)$ is hereditarily undecidable, then $\mathrm{Th}(N)$ is hereditarily undecidable as well.[1] Hence Bardakov and Tolstykh showed that $\mathrm{Th}(F)$ is hereditarily undecidable, thus answering a part of Question 4.16 in [6]. By the same argument we show that the elementary theories of all groups of Stein [4] are hereditarily undecidable.

The groups studied in [4] are defined their in terms of piecewise affine permutations of an interval. We remind these definitions. Let r be a positive real number and Λ a non-trivial subgroup of the multiplicative group $\mathbb{R}^*_+$ of positive reals. Let A be an additive subgroup of $\mathbb{R}$ containing r and invariant under the action of Λ by multiplications. Then define $\mathcal{V}(r, \Lambda, A)$ to be the group of all the bijections $x\colon [0, r) \to [0, r)$ such that:

[1] In [1] the authors state this fact with a reference to [2].

(i) x is piecewise affine with finitely many cuts and singularities;

(ii) x is right-continuous at every point;

(iii) the slope of each affine part of x is in Λ;

(iv) all cut and singular points of x, as well as their images, are in A.

Define subgroups $\mathcal{F}(r,\Lambda,A)$ and $\mathcal{T}(r,\Lambda,A)$ of $\mathcal{V}(r,\Lambda,A)$ as follows:

- $\mathcal{F}(r,\Lambda,A)$ is the subgroup of all the continuous elements of $\mathcal{V}(r,\Lambda,A)$,
- $\mathcal{T}(r,\Lambda,A)$ is the subgroup of all the elements of $\mathcal{V}(r,\Lambda,A)$ that are continuous with respect to the topology of circle (identifying $[0,r)$ with the topological quotient $[0,r]/\{0,r\}$).

The groups F, T, and V of Thompson are isomorphic to $\mathcal{F}(1,\langle 2\rangle,\mathbb{Z}[\frac{1}{2}])$, $\mathcal{T}(1,\langle 2\rangle,\mathbb{Z}[\frac{1}{2}])$, and $\mathcal{V}(1,\langle 2\rangle,\mathbb{Z}[\frac{1}{2}])$, respectively.

Theorem 1. *If G is a subgroup of $\mathcal{V}(r,\mathbb{R}^*_+,\mathbb{R})$ such that*

$$G\cap\mathcal{F}(r,\mathbb{R}^*_+,\mathbb{R})=\mathcal{F}(r,\Lambda,A),$$

then G interprets $(\mathbb{N},+,\times)$ with parameters.

(In particular, the groups $\mathcal{F}(r,\Lambda,A)$, $\mathcal{T}(r,\Lambda,A)$, and $\mathcal{V}(r,\Lambda,A)$ all interpret $(\mathbb{N},+,\times)$ with parameters.)

Corollary. *If G is as in Theorem 1, then* $\mathrm{Th}(G)$ *is hereditarily undecidable.*

Theorem 2. *If Λ is cyclic, then $\mathcal{F}(r,\Lambda,A)$ interprets $(\mathbb{N},+,\times)$ without parameters.*

Similarly to [1], Theorem 1 is proved by exhibiting in G a definable subgroup isomorphic to the restricted wreath product $\mathbb{Z}\wr\mathbb{Z}$, which is known to interpret the Arithmetic.

The interpretation without parameters constructed in the proof of Theorem 2 is entirely original.

This is a joint work with Tuna Altınel.

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The first-order theory of finitely generated fields

Bjorn Poonen

It has been conjectured that any reasonable property of finitely generated fields (e.g., "the absolute transcendence degree is odd") can be characterized by the truth of a single first-order sentence. This conjecture is still open. I will survey work towards it by F. Pop, myself, and T. Scanlon extending earlier work by J. Robinson, Yu. Ershov, and R. Rumely.

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On numbers and models, standard and nonstandard

Peter Roquette

My talk will be dedicated to the memory of Abraham Robinson (1918-1974). He was one of the great masters of our science in the last century. I will report on how I came to meet him in 1963 and about our cooperation developing during the years. This will include applications of model theory to number theory and algebra, but it should be kept in mind that his seminal ideas are not confined to these topics; he was an universalist in the true sense of the word.

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Modularity Over Imaginary Quadratic Fields

Mehmet Haluk Şengün

We discuss the (conjectural) relationship between Galois representations of imaginary quadratic fields and the so-called Bianchi modular forms. We start by overviewing the classical situation for $\mathbb{Q}$ and end the talk by presenting certain results for the imaginary quadratic fields case.

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Differential Algebra And Model Theory

Sonat Süer

We will present two results about differentially closed fields. The first one is that the generic type of the heat variety, that is, the set of solutions of the equation $\partial_1(x) = \partial_2^2(x)$, is locally modular in the sense of geometric stability theory. The second one is that the heat variety, viewed as a differential algebraic group, does not admit a normal series whose quotients are simple improperly speaking. A group is called simple improperly speaking, if it is isomorphic to all its nontrivial quotients.

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Model theory of fields of characteristic $p \neq 0$

Carol Wood

We consider fields of nonzero characteristic, specifically we study the model theory of separably closed fields in various extensions of the language of rings.

This talk will focus on separably closed fields of infinite dimension over the field of pth powers. This includes the case of a model K of DCF_p, the theory of differentially closed fields of characteristic p, and also the constant subfield C_K of K. An alternate approach using iterative Hasse derivations will be described. We discuss features of the resulting theories, and also we compare these to DCF_p.

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Quivers with potentials and their representations

Andrei Zelevinsky

This is an account of an ongoing joint work with Harm Derksen and Jerzy Weyman. We study quivers with relations given by non-commutative analogs of Jacobian ideals in the complete path algebra. This framework allows us to give a representation-theoretic interpretation of quiver mutations at arbitrary vertices. This gives a far-reaching generalization of Bernstein-Gelfand-Ponomarev reflection functors. The motivations for this work come from several sources: superpotentials in physics, Calabi-Yau algebras, cluster algebras.

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Study of clean rings: recent progress and questions

Yiqiang Zhou

An associative ring with unity is called a clean ring if each of its elements is the sum of an idempotent and a unit. This is a survey talk on the topic of clean rings. Recent work and various questions will be introduced.

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Model Theory and Noncommutative Geometry

Boris Zilber

I am going to speak about some "nonclassical" stable structures related to noncommutative geometry and mathematical physics. We treat these as topological structures in the sense which generalises Zariski topology. In order to understand the noncommutative deformation theory we introduced a (model-theoretic) notion of approximation in a class of topological structures. Some results and open questions will be discussed.

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II Contributed Talks

The discrete logarithm problem in 2-groups

Mohammad Hasan Abbaspour

Diffie and Hellman introduced public-key cryptography in 1976. The discrete logarithm problem was defined in the multiplicative group of the integers modulo a prime. This idea rapidly extended to arbitrary groups and, in particular, to elliptic curve groups.

In this paper we consider the discrete logarithm problem in 2-groups.

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On Regular Graph of Matrix Algebras

Saieed Akbari

Let R be a ring. The *total graph* of R, denoted by $T(\Gamma(R))$ is a graph with all elements of R as vertices, and two distinct vertices $x, y \in R$, are adjacent if and only if $x + y \in Z(R)$, where $Z(R)$ denotes the set of zero-divisors of

R. Let *regular graph* of R, $Reg(\Gamma(R))$, be the induced subgraph of $T(\Gamma(R))$ on the regular elements of R. Let $\mathbb{F}$ be a field. In this talk we study some graph theoretic parameters of $Reg(\Gamma(M_n(\mathbb{F}))$. Among other results, we show that the clique number of $Reg(\Gamma(M_n(\mathbb{F}))$ is finite.

Joint work with A. Fakhari and M. Jamaali

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Permutation polynomials over finite fields and generalized Stirling numbers

Esen Aksoy

This is a joint work with A. Çeşmelioğlu, W. Meidl and A. Topuzoğlu. By a classical result of Carlitz [1], S_q, the symmetric group on q letters, which is isomorphic to the group of permutation polynomials of $\mathbb{F}_q$ of degree less than $q-1$ under the operation of composition and reduction modulo $x^q - x$, is generated by the linear polynomials $ax+b$, for $a, b \in \mathbb{F}_q$, $a \neq 0$, and x^{q-2}. Consequently, as pointed out in [2], with $\mathcal{P}_0(x) = a_0 x + a_1$, any permutation of a finite field $\mathbb{F}_q$ can be represented by a polynomial

$$\mathcal{P}_n(x) = (\ldots((a_0x + a_1)^{q-2} + a_2)^{q-2} \ldots + a_n)^{q-2} + a_{n+1}, \ n \geqslant 0, \qquad (*)$$

where $a_1, a_{n+1} \in \mathbb{F}_q$, $a_i \in \mathbb{F}_q^* = \mathbb{F}_q \setminus \{0\}$ for $i = 0, 2, \ldots, n$.

Given a permutation $p(x)$ of $\mathbb{F}_q$, among those polynomials of the type $(*)$, representing $p(x)$, it is of interest to find the one with smallest n. We therefore define the *Carlitz rank* of $p(x)$, to be the smallest n satisfying $\mathcal{P}_n(x) = p(x)$ for all $x \in \mathbb{F}_q$, i.e. $p(x)$ is composed of at least n "*inversions*" x^{q-2} with n (or $n+1$) linear polynomials. In this talk, we will present a method to determine the Carlitz rank of permutations $\mathcal{P}_s$ with $s < (q-1)/2$, and also introduce a generalized form of Stirling numbers which enables us to enumerate $\mathcal{P}_n$ for $n < (q-1)/2$:

Let t, k, m be integers with $t, k \geqslant 1, m \geqslant 0$. We define $\mathcal{S}_t(k, m)$ to be the number of ways of arranging k objects into m cycles, each cycle being of length $\geqslant t$. Clearly for $t = 1$ one obtains the classical Stirling numbers of the first kind. If $\mathcal{B}(n)$ denotes the number of permutations of $\mathbb{F}_q$ with Carlitz rank n, then for all integers n with $2 \leqslant n < (q-1)/2$, we have

$$\begin{aligned}
\mathcal{B}(n) &= \sum_{m=1}^{\lfloor \frac{n+1}{3} \rfloor} \binom{q}{n+1-m} S_2(n+1-m, m)(q^2-q)(n+1-m) \\
&= \sum_{m=1}^{\lfloor \frac{n-1}{3} \rfloor} \binom{q}{n-1-m} S_2(n-1-m, m)(q^2-q)(q-n+1+m) \\
&= \sum_{m=1}^{\lfloor \frac{n}{3} \rfloor} \binom{q}{n-m} S_2(n-m, m)(q^2-q).
\end{aligned}$$

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Generalized Derivations with Invertible Values on Multilinear Polynomials

Emine Albaş

Let K be a commutative ring with unity, R a prime K-algebra of characteristic different from 2, with extended centroid C, g a non-zero generalized derivation of R, $f(x_1, ..., x_n)$ a multilinear polynomial over K in n non-commuting variables and I a non-zero two sided ideal of R. Suppose that, for $x_1, ..., x_n \in I$, $[g(f(x_1, ..., x_n)), f(x_1, ..., x_n)]$ is either zero or invertible in R. Then one of the following holds:

(i) $f(x_1, ..., x_n)$ is central-valued on R;

(ii) $g(x) = px + xq$ and $f(x_1, ..., x_n)^2$ is central-valued on R, with $p - q \in C$;

(iii) $g(x) = ax$ for all $x \in R$, where $a \in C$;

(iv) $R = D$, a division ring or $R = M_2(D)$;

(v) $g(x) = px + xp$ and $R = M_k(D)$ is a PI-ring such that $k \geqslant 3$ and $f(x_1, ..., x_n)x_{n+1}$ is an identity for any minimal right ideal of R,

where $R = M_k(D)$ is the ring of $k \times k$ matrices over a division ring D and for a suitable $p \in R$.

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Proper classes generated by submodules that have supplements

Rafail Alizade

Throughout, R is a a Dedekind domain and modules are unital R-modules. The class of all short exact sequences of modules

$$E : 0 \longrightarrow A \xrightarrow{\alpha} B \longrightarrow C \longrightarrow 0$$

such that $\operatorname{Im}\alpha$ has a supplement in B will be denoted by $\mathcal{K}$. The corresponding elements of $Ext^1_R(C,A)$ are called κ-elements (see [1]). We denote by $\mathcal{WS}upp$ the class of short exact sequences E, where $\operatorname{Im} f$ has (is) a weak supplement in B, i.e. there is a submodule K of B such that $\operatorname{Im} f + K = B$ and $\operatorname{Im} f \cap K \ll B$, and by $\mathcal{S}mall$ the class of short exact sequences E where $\operatorname{Im} f \ll B$. We show that $\mathcal{K}$ need not be a proper class of short exact sequences (see [2]) and the the proper class $\langle\mathcal{K}\rangle$ generated by $\mathcal{K}$, that is the smallest proper class containing $\mathcal{K}$, coincides with the class $\mathcal{A}bs$ of all short exact sequences, moreover we have the following proposition.

Proposition 1. $\langle\mathcal{S}mall\rangle = \langle\mathcal{K}\rangle = \langle\mathcal{WS}upp\rangle = \mathcal{A}bs$.

In the category of torsion modules $\mathcal{K}$ is a proper class and $\mathcal{K} = \langle\mathcal{S}mall\rangle$. Only projective modules are $\mathcal{K}$-projective and only injective modules are $\mathcal{K}$-injective. But the classes of coprojective and coinjective modules are not trivial. We will also discuss the global dimension of the proper class $\mathcal{C}$.

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Hereditary Radical for Near-Rings

Emin Aygün

Different type of Jacobson Radicals have been introduced for near-rings by Kaarli, Holcombe and Groenewald. In this paper, a new structure, which is called N-clogroup that possesses some interesting properties is defined. According to this structure for a near-ring N , a radical, $J_{3u}(N)$ that is closely related to the radical $J_3(N)$ is studied. It is show that $J_{3u}(N)$ is also a strongly hereditary radical.

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Extended semicommutative rings and their extensions

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Throughout this note R denotes an associative ring with identity. Recall that a ring R is called (1) *reduced* if it has no nonzero nilpotent elements; (2) *reversible* if $ab = 0$ implies $ba = 0$ for $a, b \in R$; (3) *semicommutative* if $ab = 0$ implies $aRb = 0$ for $a, b \in R$. Every reduced ring is reversible and every reversible ring is semicommutative, but the converses do not hold, in general. Another generalization of a reduced ring is an Armendariz ring. A ring R is called *Armendariz* if whenever any polynomials $f(x) = a_0 + a_1x + \cdots + a_mx^m$, $g(x) = b_0 + b_1x + \cdots + b_nx^n \in R[x]$ satisfy $f(x)g(x) = 0$, $a_ib_j = 0$ for each i and j. The Armendariz property of a ring was extended to skew polynomial rings but with skewed scalar multiplication. A ring R is called α-*Armendariz* (resp., α-*skew Armendariz*) [3] (resp., [2]) if for $p = \sum_{i=0}^{m} a_ix^i$ and $q = \sum_{j=0}^{n} b_jx^j$ in $R[x; \alpha]$, $pq = 0$ implies $a_ib_j = 0$ (resp., $a_i\alpha^i(b_j) = 0$) for all $0 \leqslant i \leqslant m$ and $0 \leqslant j \leqslant n$. Recall that an endomorphism α of a ring R is called *rigid* [4] if $a\alpha(a) = 0$ implies $a = 0$ for $a \in R$, and R is called an α-*rigid* ring if there exists a rigid endomorphism α of R.

In this note, we introduce the notion of an α-semicommutative ring with its endomorphism α, as a generalization of α-rigid rings and an extension of semicommutative rings, and study their related properties. Specially, we study the relationship between extended Armendariz rings and α-semicommutative rings. Consequently, several known results are obtained as corollaries of our results.

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A combinatorial problem in certain class of groups

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Let $\omega(x_1, ..., x_n)$ be a word in the free group of rank $n > 0$. We say that a group G is a $\mathcal{V}(\omega)$-group, if G satisfies the law $\omega(x_1, ..., x_n) = 1$, and we say that G is a $\mathcal{V}(\omega^*)$-group, if for every n infinite subsets $X_1, ..., X_n$ of G there exist $g_1 \in X_1, ..., g_n \in X_n$ such that $\omega(g_1, ..., g_n) = 1$. Longobardi et al. [8] posed the question whether every infinite $\mathcal{V}(\omega^*)$-group is a $\mathcal{V}(\omega)$-group?

The origin of this question is a problem of P. Erdös [12]. Apparently, there is no example of an infinite $\mathcal{V}(\omega^*)$-group which is not a $\mathcal{V}(\omega)$-group. In considering this question, many authors have answered the question positively for certain words (see[1, 2, 3, 4, 7, 8, 9, 10, 11, 15, 16]).

G.Endimioni proved in [7] that if ω is a word in a free group such that finiteley generated soluble groups in $\mathcal{V}(\omega)$ are polycyclic, then every finitely generated soluble group in $\mathcal{V}(\omega^*)$, belongs to the variety $\mathcal{V}(\omega)$. A. Abdollahi proved in[3] that every infinite locally soluble group of finite rank in $\mathcal{V}(\omega^*)$ belongs to the variety $\mathcal{V}(\omega)$.

In this work we prove the following generalization of the latter result of A. Abdollahi.

Theorem 1. *Let ω be a word in a free group. Then every infinite locally soluble $FASR$-group in $\mathcal{V}(\omega^*)$ belongs to the variety $\mathcal{V}(\omega)$.*

A group G is said to be a $FASR$-group, if for every abelian subgroup H of G, $r_0(H)$ and $r_p(H)$ are finite[See[14] p69].

Also we prove the following result:

Theorem 2. *Let ω be a word in a free group such that finiteley generated soluble (finite elementary abelien)-by-$\mathcal{V}(\omega^*)$ group, belongs to the variety $\mathcal{V}(\omega)$. Then every infinite locally soluble group in $\mathcal{V}(\omega^*)$ belong to $\mathcal{V}(\omega)$.*

Our notation and terminology are standard, and can be found in [14].

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Rings whose modules are weakly supplemented are perfect

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H. Bass characterized in [1] those ring R whose left R-modules have projective covers and termed them *left perfect* rings. The rings whose finitely generated left modules have projective covers are termed as *semiperfect* rings. Kasch and Mares transfered in [2] the notions of perfect and semiperfect rings to modules and characterized semiperfect modules by a lattice theoretical condition as follows: a module M is called *supplemented* if for any submodule N of M there exists a submodule L of M minimal with respect to $M = N + L$. The left perfect rings are then shown to be exactly those rings whose left R-modules are supplemented while the semiperfect rings are those whose finitely generated left R-modules are supplemented. Equivalently it is enough for a ring R to be semiperfect if the left (or right) R-module R is supplemented. A submodule

N of a module M is called *small*, denoted by $N \ll M$, if $N + L \neq M$ for all proper submodules L of M. Weakening the "supplemented" condition one calls a module *weakly supplemented* if for every submodule N of M there exists a submodule L of M with $N + L = M$ and $N \cap L \ll M$. The semilocal rings R are precisely the rings whose finitely generated left (or right) R-modules are weakly supplemented. Again it is enough that R is weakly supplemented as left (or right) R-module. By using [3, Theorem 1], we prove the following:

Theorem 1. *The following statements are equivalent for a ring R:*

(a) Every left R-module is weakly supplemented;

(b) $R^{(\mathbb{N})}$ is weakly supplemented;

(c) R is semilocal and $Rad(R^{(\mathbb{N})})$ has a weak supplement in $R^{(\mathbb{N})}$;

(d) R is left perfect.

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Envelopes and Radicals of Submodules of Free Modules

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This is a collaboration work with Assistant Professor Erol Yilmaz from Abant Izzet Baysal University.

Throughout, all rings are commutative with non-zero identity and all modules are unitary. Let R be a ring, M be an R-module and N be a proper submodule of M. If $f \in R$, then

$$N : f^{\infty} = \{m \in M : f^k m \in N \text{ for } \textit{some positive integer } k\}.$$

For any subset N of M, $E_M(N)$ is defined to be the set

$$E_M(N) = \{fm : f \in R, m \in M \quad \text{and} \quad m \in N : f^{\infty}\}.$$

This is not a submodule of M in general. The envelope of N in M is defined to be the set generated by $E_M(N)$.

Let $R = k[x_1, \dots, x_n]$ be polynomial ring over a field k and let $M = R^m$ be a free module over R for a positive integer m. In this work, we consider of the problem finding radical and envelope of a submodule N of M. A primary decomposition of a submodule N of free module $M = R^m$ will be our starting point for finding a generating set for both envelope and radical of that submodule.

In general case a primary decomposition of N in M is a representation of N as an intersection of finitely many primary submodules of M. Such a primary decomposition $N = Q_1 \cap Q_2 \cap \dots \cap Q_m$ with p_i-primary modules $Q_i \subset M$ $i = 1, \dots, m$ is said to be minimal precisely when

(i) $p_1, \dots, p_m$ are pairwise distinct, and

(ii) for all $j = 1, \dots, m$ we have $Q_j \not\supset \bigcap_{i \neq j} Q_i$.

The prime ideals in $\mathrm{Ass}(M/N)$ that are minimal with respect to inclusion are called the isolated primes of M/N, the remaining associated prime ideals are the embedded primes of M/N.

Lemma 1. *Let R be a ring and let M be an R-module. Consider a minimal primary decomposition of a submodule N of M, $N = P_1 \cap P_2 \cap \dots \cap P_s$ where P_i is p_i-primary. Let $S = \{1, 2, \dots, s\}$ and T be nonempty proper subset of S. Then*

(i) $\sqrt{N : M}M \subseteq \langle E_M(N) \rangle$

(ii) $\big(\bigcap_{i \in T} p_i\big)\big(\bigcap_{i \in S-T} P_i\big) \subseteq \langle E_M(N) \rangle$

Thus we have the following result.

Theorem 2. *Let R be a ring and let M be an R-module. Consider a minimal primary decomposition of a submodule N of M, $N = P_1 \cap P_2 \cap \dots \cap P_s$ where P_i is p_i-primary. Let $S = \{1, 2, \dots, s\}$ and T be nonempty proper subset of S. Then*

$$\langle E_M(N) \rangle = N + \sqrt{N : M}M + \sum_{T \subsetneq S} \big(\bigcap_{i \in T} p_i\big)\big(\bigcap_{i \in S-T} P_i\big)$$

Corollary 3. *If P is a p-primary submodule, then $\langle E_M(P) \rangle = P + pM$.*

Recall that the radical of N in M, denoted by $rad_M(N)$, is defined to be intersection of all prime submodules of M containing M. It is well known that $N \subseteq \langle E_M(N) \rangle \subseteq \mathrm{rad}_M(N)$. By definition of radical, one can easily show that; (i) if $N_1 \subseteq N_2$ for submodule of M, then $\mathrm{rad}_M(N_1) \subseteq \mathrm{rad}_M(N_2)$, and (ii) $\mathrm{rad}_M(\mathrm{rad}_M(N)) = \mathrm{rad}_M(N)$. The envelope of a submodule, however, does not satisfy an equation similar to second one. We can only show that

$$N \subseteq E_M(N) \subseteq E_M^2(N) = E_M(E_M(N)) \subseteq E_M^3(N) = E_M(E_M(E_M(N))) \subseteq \cdots.$$

If, for some positive integer p, $E_M^p(N) = P_1 \cap \dots \cap P_s$ is a finite intersection of prime submodules,

$$E_M^p(N) \subseteq \mathrm{rad}_M(E_M^p(N)) \subseteq \bigcap_{E_M^p(N) \subset P} P \subseteq \bigcap_{i=1}^{s} P_i = E_M^p(N)$$

Since $N \subset E^p_M(N)$, $\mathrm{rad}_M(N) \subset rad(E^p_M(N))$. Hence

$$N \subseteq E^p_M(N) \subseteq E^q_m(N) \subseteq rad_M(N)$$

implies

$$\mathrm{rad}_M(N) \subseteq \mathrm{rad}_M(E^p_M(N)) = E^p_M(N) \subseteq E^q_M(N) \subseteq \mathrm{rad}_M(\mathrm{rad}_M(N)) \\ = rad_M(N).$$

Hence $E^p_M(N) = E^q_M(N)$ for $q \geqslant p$, that means, chain terminates, and $\mathrm{rad}_M(N) = E^p_M(N)$. Therefore, in this special case, computing chain of powers of envelopes gives us radical of the submodule.

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On generalized derivation which acts as a homomorphism or an anti-homomorphism

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This is joint work with N. Argaç. Let R be a prime ring and I a non-zero right ideal of R. Let U be the left Utumi quotient ring of R and C be the center of U. Let g be a generalized derivation of R. If g acts as a homomorphism or an anti-homomorphism on I, then one of the following holds:
(i) $g(x) = ax$ for all $x \in R$ and $aI = 0$,
(ii) $g(x) = ax + xc$ for all $x \in R$ and $cI = 0$ and $(a + \lambda)I = 0$ for some $\lambda \in C$.

Throughout this paper unless specially stated, R always denotes a prime ring with center $Z(R)$, extended centroid C, left Utumi quotient ring U, and two sided Martindale quotient ring Q. For any $x, y \in R$, we set $[x, y] = xy - yx$.

By a derivation of R we mean an additive map d from R into itself satisfying the rule $d(xy) = d(x)y + xd(y)$ for all $x, y \in R$. For $b \in Q$, we denote by $ad(b)$ the inner derivation induced by b; that is , $ad(b)(x) = [b, x]$ for $x \in R$. An additive mapping $g : R \to R$ is called a generalized derivation of R if there exists a derivation d of R such that $g(xy) = g(x)y + xd(y)$ for all $x, y \in R$ [4]. Obviously any derivation is a generalized derivation. Moreover, other basic examples of generalized derivations are the following: (i) $g(x) = ax + xb$, for some $a, b \in R$; (ii) $g(x) = ax$, for some $a \in R$. Many authors have studied generalized derivations in the context of prime and semiprime rings (see [7], [4], [8]).

Let S be a nonempty subset of R and g be a generalized derivation of R. If $g(xy) = g(x)g(y)$, or $g(xy) = g(y)g(x)$ for all $x, y \in S$, then g is said to be acting as a homomorphism or an anti-homomorphism on S, respectively.

In [3], Bell and Kappe proved that if d is a derivation of a prime ring R which acts as a homomorphism or an anti-homomorphism on I, where I is a nonzero right ideal of R, then $d = 0$ on R. The aim of the present paper is to extend these results to generalized derivations.

In what follows, unless stated otherwise, R will be a prime ring. The related object we need to mention is the left Utumi quotient ring U of R (sometimes, as in [2], U is called the maximal left ring of quotients).

The definitions, the axiomatic formulations and the properties of this quotient ring U can be found in [2].

In any case, when R is a prime ring, all that we need about U is that
1) $R \subseteq U$;
2) U is a prime ring with identity;
3) The center of U, denoted by C, is a field which is called the extended centroid of R.

We also frequently use the theory of generalized polynomial identities and differential identities(see [2],[5], [6],[9]).

By motivating above results we shall prove the following theorems:

Theorem 1. *Let R be a prime ring and I be a nonzero right ideal of R. Let g be a generalized derivation on I. If g acts as a homomorphism on I, then one of the following holds:*
(i) $g(x) = ax$ for all $x \in R$ and $aI = 0$,
(ii) $g(x) = ax + xc$ for all $x \in R$, $cI = 0$ and $(a + \lambda)I = 0$ for some $\lambda \in C$.

Theorem 2. *Let R be a prime ring and I be a nonzero right ideal of R. Let g be a generalized derivation on I. If g acts as an anti- homomorphism on I, then one of the following holds:*
(i) $g(x) = ax$ for all $x \in R$ and $aI = 0$,
(ii) $g(x) = ax + xc$ for all $x \in R$, $cI = 0$ and $(a + \lambda)I = 0$ for some $\lambda \in C$.

Corollary 3 ([1], Theorem 3.4)**.** *Let R be a prime ring and g a generalized derivation of R. If g acts as a homomorphism or an anti-homomorphism on R, then $g = 0$ or g is a identity map on R.*

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Centralizers of semisimple abelian subgroups and unipotent elements in locally finite simple groups of Lie type

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The structure of the centralizers of elements in a finite simple group G give strong information about the structure of G. Brauer-Fowler's result for finite simple groups has been extended to locally finite simple groups by Hartley in [1]. In this article he also proved the following result:

Theorem 1 ([1, Theorem C]). *Let G be an infinite simple group of Lie type over an infinite locally finite field K of characteristic p and let α be an automorphism of finite order n of G. Suppose that p does not divide n. Then there exists infinitely many primes q such that α fixes an element of order q in G.*

We extend the above theorem of Hartley to d-abelian subgroups of simple locally finite groups of classical Lie type. We also give an example of a non-d-abelian subgroup such that the above result is not true.

Definition. Let G be a simple locally finite group of classical Lie type. A finite semisimple abelian subgroup A is called a **d-abelian subgroup** of G if it satisfies one of the following conditions :

(i) The root system associated with G has type A_l and $(|A|, l+1) = 1$.

(ii) The root system associated with G has type B_l, C_l or D_l and the Sylow $2-$subgroup of A is cyclic.

Our main result is the following:

Theorem 2. *Let G be a locally finite simple group of Lie type defined over an infinite locally finite field of characteristic p. Let A be a d-abelian subgroup of G. Then there exists infinitely many primes q such that A fixes an element of order q.*

Hartley proved his result for semisimple elements. We proved that this result is also true for irregular unipotent elements in classical groups of type A_l in odd characteristic. We observe that for a regular unipotent element u, the centralizer $C_G(u)$ contains only finitely many elements of distinct prime orders, so the result is not true for regular unipotent elements. When u is an irregular unipotent element, we will prove the following result for infinite locally finite simple groups of Lie type A_l:

Theorem 3. *Let $G = PSL_n(k)$ where k is an infinite locally finite field of odd characteristic and u be an irregular unipotent element. Then $C_G(u)$ contains infinitely many elements of distinct prime order.*

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Elliptic Divisibility Sequences in Certain Ranks over Finite Fields

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This is joint work with Osman Bizim. Morgan Ward in [12, 13] gave arithmetic theory of elliptic divisibility sequences and formulas for elliptic divisibility sequences with rank two and three over finite field $\mathbb{F}_p$. In this work we study elliptic divisibility sequences over finite fields.We give general terms of elliptic divisibility sequences with rank three, four, five and six over a finite field $\mathbb{F}_p$ and then we determine elliptic curves and singular curves associated with these sequences.

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Characters and Quasi-Permutation Representations of 2-Groups with $|\Omega_2(G)| = 16$

Ghodrat Ghaffarzadeh

By a quasi-permutation matrix we mean a square matrix over the complex field $\mathbb{C}$ with non-negative integral trace. Thus every permutation matrix over $\mathbb{C}$ is a quasi-permutation matrix. For a given finite group G, let p(G) denote the minimal degree of a faithful permutation representation of G (or a faithful representation of G by permutation matrices), let q(G) denote the minimal degree of a faithful representation of G by quasi-permutation matrices over the rational field $\mathbb{Q}$, and let c(G) denote the minimal degree of a faithful representation of G by complex quasi-permutation matrices. It is easy to see that

$$c(G) \leqslant d(G) \leqslant p(G)$$

where G is a finite group. In [2], finite 2-groups G have been determined with the property $|\Omega_2(G)| = 16$. We recall that $\Omega_2(G) = \langle x \in G : x^4 = 1 \rangle$. In this paper, we study characters and quasi-permutation representations of this groups which are cyclic center and have the additional property that G is non-abelian and has a normal elementary-abelian subgroup E of order 8. In [2], these groups have been determined in terms of generators and relations. Also, it is shown that G/E is either cyclic or a generalized quaternion Q_{2^n} of order 2^n, $n \geqslant 3$. If G/E is cyclic, then there exists one class of 2-group [2, theorem 3.1], and if G/E is generalized quaternion group, then there exist two classes of 2-groups [2, theorems 3.3 and 3.4].

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Combinatorial Geometries of Field Extensions

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In the fields of real and complex numbers, model theoretic algebraic closure coincides with relative field theoretic algebraic closure and form a pregeometry. The starting point for this work was the question, whether the above pregeometries i.e. $(\mathbb{C}, \mathrm{acl}_{\mathbb{Q}})$ and $(\mathbb{R}, \mathrm{acl}_{\mathbb{Q}} \cap \mathbb{R})$ (where $\mathrm{acl}_{\mathbb{Q}} \colon \mathcal{P}(\mathbb{C}) \to \mathcal{P}(\mathbb{C})$ is algebraic closure over $\mathbb{Q}$) are isomorphic. We will give a negative answer. However in this paper we consider a more general set-up. Our results are based on similar results of Evans and Hrushovski (from [1, 2]) in the case of algebraically closed fields.

We work within a large algebraically closed field $\mathfrak{C}$. By $\widehat{F}$ and $\widehat{F}^r$ we denote algebraic and purely inseparable closure of a field F in $\mathfrak{C}$. Let $K \subset L$ be an arbitrary field extension and the transcendence degree of L over K is at least 3. For $X \subseteq L$, let $\mathrm{acl}_K^L(X)$ be $\widehat{K(X)} \cap L$. We denote by $G(L/K)$ *the pregeometry* (L, acl_K^L). *The geometry* $\mathbb{G}(L/K)$ is obtained from $L \setminus \widehat{K}$ by factoring out the equivalence relation:

$$x \sim y \iff \widehat{K(x)} = \widehat{K(y)}.$$

We can also transfer the closure operation acl_K from $G(L/K)$ to $\mathbb{G}(L/K)$:

$$\mathrm{acl}_K(Y/\sim) = \mathrm{acl}_K(Y \setminus \widehat{K})/\sim.$$

Therefore we can regard the points of $\mathbb{G}(L/K)$ as sets $\mathrm{acl}_K(x)$, where $x \in L \setminus \widehat{K}$, and acl_K as the usual algebraic closure. When considering $\mathbb{G}(L/K)$ we assume that L is a perfect field and K is relatively algebraically closed in L (because $\mathbb{G}(L/K) = \mathbb{G}(\widehat{L}^r/K) = \mathbb{G}(\widehat{L}^r/\widehat{L}^r \cap \widehat{K})$).

In [1] the authors classify projective planes in $\mathbb{G}(L/K)$ for algebraically closed K and L. Using their results, we give such a classification for arbitrary fields K and L of characteristic zero. We prove a theorem about formulas with one quantifier of the first-order theory of $\mathbb{G}(L/K)$. Assume that the transcendence degree of L over K is at least 5. One of the main results of [2] is the reconstruction of the field L from $\mathbb{G}(L/K)$ when L is algebraically closed. We generalize this reconstruction to arbitrary field extension $K \subset L$ (of transcendence degree $\geqslant 5$), and thus we obtain full classification of combinatorial geometries of fields: $\mathbb{G}(L_1/K_1)$ and $\mathbb{G}(L_2/K_2)$ are isomorphic if and only if the field extensions

$$K_1 \subset L_1 \quad \text{and} \quad K_2 \subset L_2$$

are isomorphic (here we assume that L_1 and L_2 are perfect and K_1, K_2 are relatively algebraically closed). We also give a description of $\mathrm{Aut}(\mathbb{G}(L/K))$.

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Trees of Varieties over $\mathbb{Z}_p$

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To a variety V defined over $\mathbb{Z}_p$, one can naturally associate a tree: the nodes at depth n are the points $V(\mathbb{Z}/p^n\mathbb{Z})$, and the tree structure is given by the canonical maps $V(\mathbb{Z}/p^{n+1}\mathbb{Z}) \to V(\mathbb{Z}/p^n\mathbb{Z})$.

There is an old result describing cardinalities $N_n := \#\mathbb{Z}/p^n\mathbb{Z}$ (see e.g. [1]): the associated Poincaré series

$$P(T) := \sum_{n=0}^{\infty} N_n T^n$$

is a rational function in T. This obviousely means that there are strong restrictions on the trees which one can get from varieties. Thus a natural question (posed to me by Loeser) is whether one can combinatorially describe the trees which can arise, if possible in a way which is precise enough to get the rationality of the Poincaré series.

The goal of this talk is to present a conjecture which describes the possible trees very precisely and which indeed yields the desired rationality. The conjeture is true for curves. (An article is in preparation.) If there is time, I might also present some variants and generalizations. In particular, there is a variant for arbitrary definable sets in the language of valued fields.

There are several other reasons for which these trees interesting:

- If $x \in V(\mathbb{Z}_p)$ is a point of the variety, then x defines an infinite branch in the tree, and going deeper into the tree means looking at smaller neighbourhoods of x. It is not very difficult to check that if V is smooth of dimension n at x, then sufficiently close to x the tree is isomorphic to the tree of $\mathbb{A}^n$. On the other hand, if x is a singular point, then the tree close to x can get much more complicated. Thus the trees might provide a new understanding of singularities.

- For p-adic and motivic integration, it is important to understand how valuation behaves on varieties over valued fields. The trees are one way to present detailed information about the valuation, thus it is very likely that knowing the tree of a variety is helpful for the computation of such integrals.

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On some integral representations of finite groups

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Let E be a finite extension of a number field F with Galois group Γ, and let O_E and O_F be the maximal orders of E and F. Let O'_E be the intersection of valuation rings of all ramified prime ideals in the ring O_E, and let $O'_F = F \cap O'_E$. Denote $\phi_E(t) = [E(\zeta_t) : E]$ where ζ_t is a primitive t-root of 1. Let $F(G)$ be a field obtained via adjoining to F all matrix coefficients of all matrices $g \in G \subset GL_n(E)$.

Theorem 1.

(i) *For a given number field F and integers n and t, there is only a finite number of normal extensions E/F such that $E = F(G)$ and G is a finite abelian Γ-stable subgroup of $GL_n(O_E)$ of exponent t.*

(ii) *For a given number field F and integers n and $d = [E\colon F]$, there is only a finite number of fields $E = F(G)$ for some finite Γ-stable subgroup G of $GL_n(O_E)$.*

Theorem 2. *Let $d > 1, t > 1$ and $n \geqslant \phi_E(t)d$ be given integers, and let E/F be a given extension of degree d. Then there is an abelian Γ- stable subgroup $G \subset GL_n(E)$ of exponent t such that $E = F(G)$.*

Theorem 3. *Let $d > 1, t > 1$ be given rational integers, and let E/F be an unramified extension of degree d.*

(i) *If $n \geqslant \phi_E(t)d$, there is a finite abelian Γ- stable subgroup $G \subset GL_n(O'_E)$ of exponent t such that $E = F(G)$.*

(ii) *If $n \geqslant \phi_E(t)dh$ and h is the exponent of the class group of F, there is a finite abelian Γ-stable subgroup $G \subset GL_n(O_E)$ of exponent t such that $E = F(G)$.*

(iii) *If $n \geqslant \phi_E(t)d$ and h is relatively prime to n, then G given in 1) is conjugate in $GL_n(F)$ to a subgroup of $GL_n(O_E)$.*

(iv) *If d is odd, then G given in 1) is conjugate in $GL_n(F)$ to a subgroup of $GL_n(O_E)$.*

In all cases above G can be constructed as a group generated by matrices $g^\gamma, \gamma \in \Gamma$ for some $g \in GL_n(E)$.

Theorem 4. *Let E/F be a given extension of degree d, and let $G \subset GL_n(E)$ be a finite abelian Γ-stable subgroup of exponent t such that $E = F(G)$ and n is the minimum possible. Then $n = d\phi_E(t)$ and G is irreducible under conjugation in $GL_n(F)$. Moreover, if G has the minimal possible order, then G is a group of type $(t, t, ..., t)$ and order t^m for some integer $m \leqslant d$.*

The case $F = \mathbf{Q}$, the field of rationals, is specially interesting. The following theorem was proved jointly by H.-J. Bartels and the author using the classification of finite flat group schemes over $\mathbf{Z}$ annihilated by a prime p obtained by V. A. Abrashkin and J.- M. Fontaine:

Theorem 5. *Let $K/\mathbf{Q}$ be a normal extension with Galois group Γ, and let $G \subset GL_n(O_K)$ be a finite Γ-stable subgroup. Then $G \subset GL_n(O_{K_{ab}})$ where K_{ab} is the maximal abelian over $\mathbf{Q}$ subfield of K.*

Similar results for totally real extensions $K/\mathbf{Q}$ were considered by the author earlier. In this case there are some interesting arithmetic applications to positive definite quadratic lattices and Galois cohomology.

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Cofinitely coclosed submodules

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Let R be an arbitrary ring with unity. Take all modules to be *left* R-modules. The motivation for us to introduce cofinitely coclosed submodules comes from *relative homological algebra* (when we consider *proper classes* related with supplements) and investigation of the property $\mathrm{Rad}V = V \cap \mathrm{Rad}M$ for a submodule V of M:

(i) For a submodule V of a module M, V is said to be a *coneat* submodule if every module N with $\mathrm{Rad}N = 0$ is injective with respect to the inclusion $V \hookrightarrow M$. It turns out that V is a coneat submodule of M if and only if V is a *Rad-supplement* of a submodule U of M in M, that is, $U + V = M$ and $U \cap V \subseteq \mathrm{Rad}V$. So every supplement submodule is always a coneat submodule (but of course the converse does not hold in general). See [5], [3], [1, 1.11] and [4, 10.11].

(ii) By the above definition, the proper class *injectively generated* by all modules with zero radical is the class of all short exact sequences $0 \to A \to B \to C \to 0$ of modules such that the image of the monomorphism $A \to B$ is coneat in B. Another larger proper class that is natural to consider is the proper class injectively generated by all *simple* modules.

(iii) A submodule K of M is called *small* in M (denoted by $K \ll M$) if $M \neq K + T$ for every proper submodule T of M. Given submodules $K \leqslant L \leqslant M$, the inclusion $K \leqslant L$ is called *cosmall in* M if $L/K \ll M/K$ (see [4, 3.1]). A submodule $L \leqslant M$ is called *coclosed in* M if L has *no* proper submodule K for which the inclusion $K \leqslant L$ is cosmall in M (see [4, 3.6]).

(iv) The property $\mathrm{Rad}V = V \cap \mathrm{Rad}M$ for a submodule V of M holds if V is a supplement in M and moreover if V is coclosed in M (see [7, 41.1] and [4, 3.7]). This property also holds when V is a Rad-supplement in M; see [6, 6.1.7].

(v) Cofinite submodules introduced in [2] is a natural concept that is used and behaves well: A submodule N of a module M is said to *cofinite* if M/N is finitely generated.

We shall mention the following main results and some other related properties of cofinitely coclosed submodules.

Theorem 1. *For a submodule $N \subseteq M$ the following are equivalent.*

(i) *For* no *proper* cofinite *submodule K of N, the inclusion $K \leqslant N$ is cosmall in M (i.e. $N/K \ll M/K$).*

(ii) *For* no maximal *submodule K of N, the inclusion $K \leqslant N$ is cosmall in M (i.e. $N/K \ll M/K$).*

(iii) *If K is a maximal submodule of N, then there exists a maximal submodule L of M such that $K = N \cap L$.*

A submodule $N \subseteq M$ satisfying any of the above equivalent conditions is said to be ***cofinitely coclosed*** in M.

Theorem 2. *The proper class injectively generated by all* simple *modules equals the proper class of all short exact sequences $0 \to A \to B \to C \to 0$ of modules such that the image of the monomorphism $A \to B$ is* cofinitely coclosed *in B.*

Theorem 3. *For a submodule N of an R-module M, consider the following properties:*

(i) N *is* coneat *in* M.

(ii) N *is* cofinitely coclosed *in* M.

(iii) $\mathrm{Rad}N = N \cap \mathrm{Rad}M$.

We always have (i) $\Longrightarrow$ (ii) $\Longrightarrow$ (iii).
If R is a semilocal ring, then all are equivalent.
And conversely, if all are equivalent for every submodule N of every module M, then R is necessarily semilocal.

Joint work with Engin Büyükaşık (İzmir Institute of Technology).

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Model-theoretic behaviour of certain equations in groups

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Abstract: The study of equations in groups is a difficult task which has been evolving from various points of view, combinatorial group theory on the one hand and geometric group theory on the other hand, with in the latter case the voluminous work of Sela on definable sets in free groups. One of the conclusions of this works is the model-theoretic stability of free groups. In particular definable sets in pairs of variables x and y in free groups are stable, which means that there exists a uniform bound n for which the definable set encodes a maximal chain of subsets of a set of cardinal n. Such a bound is then called the ladder index of the definable set in question, or equivalently of the formula defining it.

In this talk we will consider the ladder index of certain important group equations in certain basic group operations such as free products. We will notably see that stability is preserved in some specific cases, with a well understood control of the ladder index. The motivation is to prevent certain wild phenomena discovered in [2] such as Shelah's Independence Property of most group equations in existentially closed CSA-groups (an axiomatic generalization of free groups and more generally of torsion-free Gromov-hyperbolic groups), and notably to force partial stability in this context.

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Amply Weak Radical Supplemented Modules

Burcu Nişancı

This is joint work with Ergül Türkmen and Ali Pancar. Let R be a ring and M be a left R-module. M is called an amply weak radical supplemented module in case $M = U + V$ implies that U has a weak Rad-supplement $L \subseteq V$. In this paper various properties of this module is developed and it is shown that R is semilocal if and only if every left R-module is amply weak Rad-supplemented.

Throughout this paper we assume that R is a ring with identity and all modules are unital left R - modules. Let M be an R - module. A submodule U of M is called small in M, written as $U << M$, if, for every submodule K of M, the equality $U + K = M$ implies $K = M$. By $\operatorname{Rad} M$ we denote the sum of all small submodules of M or, equivalently the intersection of all maximal submodules of M. Let M be a module and U, V be submodules of M. A submodule V of M is called radical supplement or briefly Rad-supplement of U in M if $U + V = M$ with $U \cap V \subseteq \operatorname{Rad} V$ (see [1], Theorem 10.14]). A module M is Rad-supplemented if every submodule U of M has a Rad-supplement in M. Let U be a submodule of M. A submodule V of M is called a weak Rad-supplement of U in M if $U + V = M$ and $U \cap V \subseteq \operatorname{Rad} M$. A module M is weakly Rad -supplemented if every submodule has a weak Rad -supplement in M [2]. Every Rad -supplemented module is a weakly Rad-supplemented module, but a weakly Rad-supplemented module need not to be Rad-supplemented [2].

Let M be an R-module and U any submoule of M. We call M as amply weak Rad-supplemented if every submodule V of M such that $M = U + V$ contains a weak Rad-supplement of U in M. Clearly every amply weak Rad-supplemented module is weakly Rad-supplemented.

Lemma 1. *Let M be an R-module and V be a weak* Rad-*supplement of U in M. If $L \subseteq U$, then $(V + L)/L$ is a weak* Rad-*supplement of U/L in M/L.*

Proposition 2. *Let M be an R-module. If M is amply weak* Rad-*supplemented, then every factor module of M is also amply weak* Rad-*supplemented.*

Proposition 3. *If every submodule of M is weakly* Rad-*supplemented, M is an amply weak* Rad-*supplemented module.*

Theorem 4. *A ring R is semilocal if and only it every left R-module is an amply weak* Rad-*supplemented module.*

Theorem 5. *Let M be an R-module. If M is a π-projective weakly* Rad-*supplemented module, then M is an amply weak* Rad-*supplemented module.*

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Groups Representations and Terminal Control of Spatial Rotations

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This is joint work with Alexander Milnikov.

The control of spatial rotations of mechanical systems (manipulators, joint mechanisms and others) demands the calculation of Euler angles which realize the given rotation. This problem is actually the central one in the theory of manipulating robots and spatial joint mechanisms[1–5]. However in many cases the existing methods of its solution are fraught with essential computational difficulties. The reason of it is most likely the widely used method of representation of three-dimensional rotations: the so-called basic representation of a group of spatial rotations [6,7]. However this method has a number of disadvantages. One of these disadvantages is, firstly, that the problem of obtaining such a matrix for a concrete rotation is a difficult task in itself because its solution demands the calculation of a product of several matrices of elementary rotations about the coordinate axes and, secondly, that, Euler angles cannot be expressed in terms of the functions of the coordinates of three points—central, initial and terminal—which define the considered rotation. The latter circumstance leads to a necessity to formulate the technological task in terms of angles of rotation about the coordinate axes that creates additional problems.

To cope with the above-mentioned disadvantages, the notion of three-dimensional generalized rotations has been introduced and on the base of 1/2 weight representation of three-dimensional rotations group [8] the spinor model of said rotations has been elaborated.

Relations between the parameters of the spinor representation of a group of three-dimensional generalized rotations and the coordinates of the initial and terminal points of rotation are obtained. Simple relations between the elements of a three-dimensional orthogonal matrix of the basic representation and the Euler angles, on the one hand, and the coordinates of the initial and terminal points of rotation, on the other hand were derived [9,10]. The spinor method of solution of an inverse kinematic problem for spatial mechanisms with spherical pairs was elaborated and the corresponding algorithm has proposed. On the base of spinor representation of spatial rotations, simple formulas are obtained for calculation of controlling Euler angles [11,12].

The obtained results have enabled us to reduce the actually three-dimensional problem of spatial motion control to the one-dimensional problem. A general variational method is obtained to solve problems of terminal control of spatial rotations. Simple and reliable adaptive algorithms are obtained, by means of which various partial problems on the terminal control of acceleration, transfer of the object to a given point, and approach are solved under various terminal conditions. New algorithms of control of spatial rotations of manipulating robots are suggested [13].

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Duo Modules

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A submodule N of an R-module M is called *fully invariant* if $f(N) \subseteq N$ for any endomorphism f of M. M is called a *(weak) duo module* provided every (direct summand) submodule of M is fully invariant. We investigate some properties of (weak) duo modules and it is proved that if R is a commutative domain with field of fractions K then a torsion-free uniform R-module is a duo module if and only if every element k in K such that kM is contained in M belongs to R. Moreover every non-zero finitely generated torsion-free duo R-module is uniform. In addition, if R is a Dedekind domain then a torsion R-module is a duo module if and only if it is a weak duo module and this occurs precisely when the P-primary component of M is uniform for every maximal ideal P of R. *Joint work with A. Harmancı and P.F. Smith*

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Rad-supplemented modules

Salahattin Özdemir

Let R be an arbitrary ring with unity. Take R-modules to be *left* R-modules. R-$\mathcal{M}od$ denotes the category of *left* R-modules.

We will see some properties of Rad-supplemented modules and in general τ-supplemented modules where τ is a radical for R-$\mathcal{M}od$. The motivation for considering Rad-supplements (=coneat submodules) and τ- supplements in general comes from *relative homological algebra* as explained in the next paragraphs when we consider *proper classes* related with supplements. One of the main questions we shall answer is when are all left R-modules Rad-supplemented. In the investigation of this problem, the notion of radical modules, reduced modules and coatomic modules turn out to be useful (see [10]). In these definitions, instead of Rad, we can use any (pre)radical τ for R-$\mathcal{M}od$ which will be useful in the investigation of the properties of τ-supplemented modules. For a module M, denote by $P(M)$ the sum of all submodules U of M such that $\operatorname{Rad} U = U$. For the ring R, $P(R)$ is a two-sided ideal.

Neat subgroups of abelian groups has been generalized to modules in [7, 8]: A monomorphism $f : K \to L$ of modules is called *neat* if each simple module S is *projective* with respect to the projection $L \to L/\operatorname{Im} f$, that is, the Hom sequence $\operatorname{Hom}(S, L) \to \operatorname{Hom}(S, L/\operatorname{Im} f) \to 0$ is exact. Dually, the class of coneat submodules has been introduced in [2] and [5]: A monomorphism $f : K \to L$ of modules is called *coneat* if each module M with $\operatorname{Rad} M = 0$ is *injective* with respect to it, that is, the Hom sequence $\operatorname{Hom}(L, M) \to \operatorname{Hom}(K, M) \to 0$ is exact. In [8], it is pointed out that supplement submodules induce a proper class of short exact sequences (see also [4]). Coneat monomorphisms induce a proper class injectively generated by modules with zero radical and this proper class contains the proper class induced by supplement submodules. In the definition of coneat submodules, we can take any radical τ for R-$\mathcal{M}od$ instead of Rad. The characterization of coneat submodules as Rad-supplements is the particular case $\tau = \operatorname{Rad}$ in the following theorem:

Theorem 1. *(see [1, 1.11] or [3, 10.11]) Let τ be a radical for R-$\mathcal{M}od$. For a submodule $V \leqslant M$, the following statements are equivalent:*

(i) Every module N with $\tau(N) = 0$ is injective with respect to the inclusion $V \hookrightarrow M$;

(ii) there exists a submodule $U \leqslant M$ such that $U + V = M$ and $U \cap V = \tau(V)$;

(iii) there exists a submodule $U \leqslant M$ such that $U + V = M$ and $U \cap V \leqslant \tau(V)$.

If these conditions are satisfied, then V is called a τ-supplement *in M.*

Note that the last condition in the previous proposition likes being a supplement condition except that instead of $U \cap V \ll V$ ($U \cap V$ is small in V), the condition $U \cap V \leqslant \tau(V)$ is required. The usual definitions are then given as follows: For submodules U, V of a module M, V is said to be a *τ-supplement* of U in M if $U + V = M$ and $U \cap V \leqslant \tau(V)$. A module M is called *τ-supplemented* if every submodule of M has a τ-supplement. See [1] and [3] for some properties of τ-supplements and τ-supplemented modules. For $\tau = \operatorname{Rad}$, this gives the definition of *Rad-supplement* of a submodule and *Rad-supplemented* module. See also [9]; Rad-supplemented modules are called *generalized supplemented* modules there. See [6, Ch.6] for a survey of related results on Rad-supplemented modules. Our main result is the following:

Theorem 2. *(i) Every left R-module is Rad-supplemented iff $R/P(R)$ is left perfect.*

(ii) If R is a left duo ring, that is, all of its left ideals are two-sided ideals, then R is Rad-supplemented iff $R/P(R)$ is semiperfect.

Over Dedekind domains, the structure of supplemented modules is completely determined in [10]. Using this we obtain:

Theorem 3. *If R is a Dedekind domain, then an R-module M is Rad-supplemented if and only if M/D is supplemented, where $D = P(M)$ is the divisible part of M.*

Joint work with: Engin Büyükaşık (İzmir Institute of Technology) and Engin Mermut (Dokuz Eylül University)

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Diagram mutations and quasi-Cartan matrices

Ahmet Seven

Diagram mutations are a class of graph transformations introduced by Fomin and Zelevinsky to define cluster algebras. Quasi-Cartan matrices give a generalization of Cartan matrices that define Kac-Moody algebras. In this talk we will discuss how those two notions are related. In particular, we will discuss how quasi-Cartan matrices can be used to study mutation classes of graphs.

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Modules which lie above a supplement submodule

Nurhan Sökmez

This is joint work with Celil Nebiyev and Berna Koşar. In this work some properties of supplement submodules and lying above are investigated. Let M be an R-Module. Then every submodule of M lies above a supplement submodule in M if and only if M is amply supplemented.

In this work R will denote an arbitrary associative ring with unity and all modules are unital left R-modules. Let M be an R-module. A submodule S is called a small submodule of M if for every proper submodule A of M, $M \neq A + S$. We will use the notations $S << M$ to indicate that a submodule S is a small in M.

Let M be an R-module. Let N be a submodule of M. A supplement of N in M is a submodule K of M minimal with respect to the property $M = N + K$, equivalently, $M = N + K$ and $N \cap K << K$. A submodule K of M is called a supplement submodule in M provided there exists a submodule N of M such that K is a supplement of N in M. If every submodule of M has a supplement in M, then M is called supplemented.

Any module M is called amply supplemented if for any two modules A and B of M with $M = A + B$, there exists a supplement P of A in M which is contained in B. Clearly, every amply supplemented module is supplemented.

Let M be an R-module and $K \leqslant U \leqslant M$. If $U/K << M/K$ then we say U lies above K. We know that, U lies above a submodule K of M if and only if $K \leqslant U$ and for every submodule T of M with $U + T = M$, then $K + T = M$.

Let M be an R-module. M satisfies (D_1) if for every submodule N of M there exist submodules K and K' of M such that $M = K \oplus K'$, $K \leqslant N$ and $N \cap K' << K'$. Furthermore M satisfies (D_1) iff every submodule of M lies above a direct summand of M.

Results

Lemma 2.1. Let M be an R-module. If K is a supplement submodule in M, then K is a supplement submodule in every $L \leqslant M$ which $K \leqslant L$.

Lemma 2.2. Let M be an R-module and V be a supplement submodule in M. Then every submodule which is a supplement submodule in V is a supplement submodule in M.

Corollary 2.3. Let M be an R-module, V be a supplement submodule in M and $K \leqslant V$. Then K is a supplement submodule in V if and only if K is a supplement in M.

Lemma 2.4. Let M be a module. Suppose that every submodule of M lies above a supplement submodule of M. Then every submodule of M have this property.

Lemma 2.5. Let M be a module. If every submodule of M lies above a supplement submodule of M, then M is supplemented.

Lemma 2.6. The following are equivalent for any module M.

(1) Every submodule of M lies above a suplement submodule in M.

(2) For any submodule U of M, there is a suplement submodule Y of M with $Y \subset U$,$U = Y + Y'$ and $Y' << M$.

Lemma 2.7. The following are equivalent for any R-module M.

(1) Every submodule of M lies above a suplement submodule of M.

(2) For any submodule U of M, Y is supplemented with $Y \subset U$,$U = Y + Y'$ and $Y' << M$.

Theorem 2.8. The following are equivalent for any R-module M.

(1) M is amply supplemented.

(2) Every submodule of M lies above a suplement submodule of M.

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The Number of Rational Points on Elliptic Curves Related to Binary Quadratic Forms

Ahmet Tekcan

This is joint work with Arzu Özkoç. Let p be a prime number such that $p \equiv 1 (\mathrm{mod}\, 4)$, let $\mathbb{F}_p$ be a finite field, let $F(x, y) = x^2 + xy - \frac{p-1}{4} y^2$ be the principal indefinite binary quadratic form of discriminant $\Delta = p$ and let $E : y^2 = x^3 + x^2 - \frac{p-1}{4} x$ be the corresponding elliptic curve over $\mathbb{F}_p$. In this work, we considered the number of rational points on E. We proved that the order of E over $\mathbb{F}_p$ is p if $p \equiv 1(\mathrm{mod}\, 8)$ or $p + 2$ if $p \equiv 5(\mathrm{mod}\, 8)$. Further we derived some formulas on the sum of $x-$and $y-$coordinates of all points (x, y) on E. Later we considered the reduction of F. We proved that the reduction of F is $\rho^2(F) = (1, 1+2j, j^2 + j - 2k)$ if $p \equiv 1(\mathrm{mod}\, 8)$ or $\rho^2(F) = (1, 1 + 2j, j^2 + j - 1 - 2k)$ if $p \equiv 5(\mathrm{mod}\, 8)$, where $k = \frac{p-1}{8}$ or $\frac{p-5}{8}$, respectively and j is any positive integer such that $k \in A_j$. Finally, we considered the number of rational points on elliptic curves E^{ρ^2} corresponding the forms $\rho^2(F)$.

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Extensions of Weakly Supplemented Lattices

S. Eylem Toksoy

L will mean a complete modular lattice with smallest element 0 and greatest element 1. An element e in a lattice L is called *essential* if $e \wedge a \neq 0$ for every non-zero element a in L. The set of essential elements of L is denoted by $E(L)$ (see [3]). An element c is called a *pseudo-complement* of an element b in L if $b \wedge c = 0$ and c is maximal with respect to this property. L is said to be *pseudo-complemented* if every element of L has a pseudo-complement in L. An element b is called an E-complement of an element a of L if $a \wedge b = 0$ and $a \vee b \in E(L)$. A lattice L is called E-complemented if every element of L has an

E-complement in L. Pseudo-complemented lattices are E-complemented. It is well known in module theory that every module is pseudo-complemented. We give an example to show that this is not true in lattice theory.

A lattice L is said to be *supplemented (*respectively, *weakly supplemented)* if every element a of L has a supplement (respectively, weak supplement) in L, i.e. an element b such that $a \vee b = 1$ and $a \wedge b \ll b/0$ (respectively, $a \wedge b \ll L$).

The following Lemma was shown by Alizade and Büyükaşık (see [2], Lemma 2.1) by using elements of modules, so it can not be directly generalized to lattice theory. We generalize this lemma by using lattice theoretic methods. Also we show by an example that this lemma need not be true for lattices that are not compactly generated.

Lemma 1. *Let L be a compactly generated lattice (see [4]) and a be a cofinite element of L. If b is a weak supplement of a in L, then a has a weak supplement c in L such that $c \leqslant b$ and c is compact.*

Theorem 2. *If $1/a$ and $a/0$ are weakly supplemented and a has a weak supplement in L, then L is also weakly supplemented.*

Joint work with: Refail Alizade, İzmir Institute of Technology

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On the automorphism groups of infinitely generated free nilpotent groups

Vladimir Tolstykh

A long-standing conjecture due to Baumslag states that the automorphism tower over any finitely generated nilpotent group terminates after finitely many steps. The conjecture for the case of finitely generated free nilpotent groups has been confirmed in 2003 by Kassabov [2]. Kassabov generalized the result by Dyer of Formanek [1] stating that the height of the automorphism tower over a f.g. free nilpotent group N of class 2 is two (and hence the group $\mathrm{Aut}(N)$ is complete) except for the case when N is a 3-generator (in this case the height of the automorphism tower is three). In [3] the result by Dyer and Formanek has been transferred to infinitely generated free nilpotent groups of class 2: it turned out that the automorphism groups of infinitely generated free nilpotent groups of class 2 are also complete. In our talk we shall discuss some results towards generalizing the latter result to infinitely generated free nilpotent groups of arbitrary class $\geqslant 2$.

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Cofinitely Radical Supplemented Modules

Ergül Türkmen

This is joint work with Ali Pancar, Burcu Nişancı and Figen YüzbaşıLet R be a ring and M be a left R-module. M is called cofinitely Rad-supplemented module or briefly a crs-module if every cofinite submodule has a Rad-supplement in M. We show that M is crs-module iff every maximal submodule of M has a Rad-supplement in M and we give some properties of crs-module.

Let M be a module and U, V be submodules of M. A submodule V of M is called radical supplement or briefly Rad-supplement of U in M if $U + V = M$ with $U \cap V \subseteq \operatorname{Rad} V$ (see[2], Theorem 10.14]). A module M is Rad-supplemented if every submodule U of M has a Rad-supplement in M.

A submodule U of M is called cofinite (in M) if the factor module M/U is finitely generated [1]. We call M as a cofinitely Rad-supplemented or briefly a crs-module if every cofinite submodule has a Rad-supplement in M.

Lemma 1. *Let M be an R-module and V be a* Rad*-supplement of U in M. If $L \subseteq U$, then $(V + L)/L$ is a* Rad*-supplement of U/L in M/L.*

Proposition 2. *Let M be a crs-module and $N \subseteq M$ with cofinite. Then M/N is* Rad*-supplemented.*

Lemma 3. *Let N and U be submodules of M with crs-module N and U is cofinite. If $N + U$ has a* Rad*-supplement in M, then U has also a* Rad*-supplement in M.*

Proposition 4. *An arbitrary sum of crs-modules is a crs-module.*

Let M be an R-module. By $\operatorname{crs}(M)$ we will denote the sum of all crs-submodules of M. Clearly $\operatorname{Crs}(M)$ is a crs-module.

Lemma 5. *Let M be an R-module. If M has a unique maximal submodule, then M is a crs-module.*

Theorem 6. *The following statements are equivalent for an R-module M.*

(i) *M is a crs-module.*

(ii) *Every maximal submodule of M has a* Rad-*supplement in M.*

(iii) *$M/Crs(M)$ does not contain a maximal submodule.*

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Simple Groups of Finite Morley Rank with a Tight Automorphism whose Centralizer is Pseudofinite

Pınar Uğurlu

A group (field) is called *pseudofinite* if it is an infinite model of the theory of finite groups (fields). In other words, a *pseudofinite group (field)* is an infinite group (field) which is elementarily equivalent to a non-principal ultraproduct of finite groups (fields). Simple pseudofinite groups are classified by John S. Wilson [8] up to elementary equivalence. Wilson proved that every simple pseudofinite group is elementarily equivalent to a Chevalley group over a pseudofinite field by using the classification of finite simple groups.

In this talk, firstly, we will consider definably simple pseudofinite groups (that is, groups that have no non-trivial proper definable normal subgroup) with descending chain condition on centralizers and show that Wilson's proof can be adapted to prove that such a group is elementarily equivalent to a Chevalley group over a pseudofinite field.

Then we will deal with pseudofinite structures arising in finite Morley rank context as fixed points of a tight automorphism. An automorphism α of an infinite simple group of finite Morley rank is called *tight* if the following conditions hold:

- α maps definable sets to definable sets.
- If $H < G$ is a connected definable α-invariant subgroup of G, then

$$d(C_H(\alpha)) = H$$

As we are working in finite Morley rank context, descending chain condition on centralizers is satisfied. Therefore, the result above is applicable and by using it we will sketch the proof of the following:

Theorem. *Let G be an infinite simple group of finite Morley rank and α a tight automorphism of G. Assume that $C_G(\alpha) = P$ is pseudofinite. Then there is a definable (in P) normal subgroup S of P such that $S \trianglelefteq P \leqslant Aut(S)$ where S is elementarily equivalent to a Chevalley group over a peseudofinite field.*

These results have been obtained on the way to prove the following conjecture suggested by Alexandre Borovik as my PhD project:

Conjecture. *Let G be an infinite simple group of finite Morley rank and α be a tight automorphism of G. Assume that $C_G(\alpha)$ is pseudofinite. Then G is a simple algebraic group over an algebraically closed field, and α is a non-standard Frobenius automorphism.*

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Computing Radicals of Submodules of Free Modules over Polynomial Rings

Erol Yılmaz

This is a joint work with Sibel Kılıçarslan Cansu.

Throughout, all rings are commutative with non-zero identity and all modules are unitary. Let R be a ring and let M be an R-module. A proper submodule N of M is called p-prime (resp. p-primary) if $rm \in$ for $r \in R$ and $m \in M$ implies that either $m \in M$ or $r \in p = N : M$ (resp. $m \in M$ or $r \in p = \sqrt{N : M}$). The radical of N in M, denoted by $rad_M(N)$, is defined to be intersection of all prime submodules of M containing M, or $rad_M(N) = M$ in case no prime submodule of M contains N.

Let $R = k[x_1, \ldots, x_n]$ be polynomial ring over a field k and let $M = R^m$ be a free module over R for a positive integer m. In this paper, we consider of the problem finding radical and envelope of a submodule N of M. Although a closely related problem that of finding primary decomposition of a submodule in this setting has been extensively studied (for example see [1] and [2]), it seems to be there is no method developed for finding a generating set of radical or envelope of a submodule. We also mention that a characterization of elements of radical of a submodule of a free module is given in [3].

A primary decomposition of a submodule N of free module $M = R^m$ will be our starting point for finding a generating set for radical of that submodule.

Definition. For an arbitrary submodule a primary decomposition of N in M is a representation of N as an intersection of finitely many primary submodules of M. Such a primary decomposition $N = Q_1 \cap Q_2 \cap \cdots \cap Q_m$ with p_i-primary modules $Q_i \subset M (i = 1, \ldots, m)$ is said to be minimal precisely when
(a) $p_1, \ldots, p_m$ are pairwise distinct, and
(b) for all $j = 1, \ldots, m$ we have

$$Q_j \not\supset \bigcap_{i \neq j} Q_i.$$

The prime ideals in $\mathrm{Ass}(M/N)$ that are minimal with respect to inclusion are called the isolated primes of M/N, the remaining associated prime ideals are the embedded primes of M/N. A submodule N is called quasi-p-primary submodule in R-module M if N has a unique isolated prime p.

Definition. Let R be a ring and N be a submodule of an R-module M. For any prime ideal p of R, the saturation of $N + pM$ defined as

$$S_p(N + pM) := \{m \in M : cm \in N + pM \text{ for some } c \in R \setminus p\}.$$

Proposition 1. (cf. [4], Proposition 4.1) *Let $M = R^m$ be a free module over a ring R and N be a proper submodule of M and let $p \supseteq N : M$ be a prime ideal of R.*
(i) $S_p(N + pM) \neq M$
(ii) $S_p(N + pM)$ is p-prime submodule.

Proposition 2. *Let M and N be defined as above.*
(i) If K is a p-prime submodule such that $N \subseteq K$, then $S_p(N + pM) \subseteq K$.
(ii) As a result of (i),

$$\mathrm{rad}_M(N) = \bigcap_{p \supseteq N:M} S_p(N + pM).$$

Corollary 3. ([4], Corollary 5.6) *If M is a Noetherian R-module and N is a proper submodule of M,then there exists a positive integer n and prime ideals $\{p_1, \ldots, p_n\}$ where $p_i \supseteq N : M$ for each i such that*

$$\mathrm{rad}_M(N) = \bigcap_{i=1}^{n} S_{p_i}(N + p_i M)$$

Theorem 4. *Let N, M and p be defined as above. Then $N + pM$ is quasi-p-primary. If $Q_1, Q_2, Q_3 \ldots$ are p-primary components of $N + pM, Q_1 + pM, Q_2 + pM, \ldots$ respectively, then there exits a positive integer k for which $S_p(N + pM) = Q_k$.*

Theorem 5. *The following algorithm compute the radical of a submodule.*
Input : A submodule N of M.
Output: $\mathrm{rad}_M(N)$.
$\mathrm{rad}_M(N) := M$
$\mathrm{Ass} := \mathrm{Ass}(M/N)$
WHILE ($\mathrm{Ass} \neq \emptyset$)
Select $p \in \mathrm{Ass}$
Compute $S_p(N + pM)$
$\mathrm{rad}_M(N) := \mathrm{rad}_M(N) \cap S_p(N + pM)$
$\mathrm{Ass} := \mathrm{Ass} \setminus \{p\} \cup \mathrm{Ass}(M/(N + pM))$
ENDWHILE

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III Posters

The Relationship Among Elliptic Divisibility Sequences, Elliptic Curves and Binary Quadratic Forms

Osman Bizim

This is joint work with Betül Gezer, Arzu Özkoç, and Ahmet Tekcan. In this work, we derive a relation among elliptic sequences (h_n), elliptic curves E and binary quadratic forms F, that is,

$$\{(h_n)\} \to \{E\} \to \{F\}$$

Our aim is to combine to topics of mathematics such as elliptic divisibility sequences, elliptic curves, binary quadratic forms and also quadratic congruences over finite fields. Ward in [23], show that we can associated to each elliptic divisibility sequences an elliptic curve, and hence determine the relation between these two aspects. In this work, by naming all elliptic divisibility sequences determined the same elliptic curve as elliptic congruence sequences, we have found families of congruence sequences for primes $p = 5$ and $p = 7$. After that by passing, to quadratic forms from the elliptic curves associated to these equivalence classes, the fundamental properties of quadratic forms associated to elliptic curves. The first part of this work is for the basis notations regarding these three concepts. On the second chapter, elliptic curves associated to elliptic divisibility sequences, and on the third chapter, some properties of binary quadratic forms associated to these elliptic curves are given.

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Strongly prime near-ring modules

N. J. Groenewald

The notion of a strongly prime module is introduced for near-rings. We show that the strongly prime radical of a near-ring can be characterized in terms of faithful strongly prime nearing modules. The notion of a special class of near-ring modules is introduced and it is proved that it gives rise to special near-ring radical classes.

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Integrally Indecomposable Polytopes

Fatih Koyuncu

Gao gave a criterion for the integral indecomposability, with respect to Minkowski sum, of polytopes lying inside a pyramid with an integrally indecomposable base. In this study, we strengthened this criterion for the polytopes lying inside the convex hull of two polytopes, one of which is integrally indecomposable, being in two parallel nonintersecting hyperplanes.

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IFP N-groups and certain classes of near-rings

A. Sezgin

This is joint work with A. O. Atagün and F. Taşdemir. In this paper, two new types of N-groups when N is a near-ring are introduced and these are illustrated by some examples. Surprisingly, these two notions are seen naturally, when N is in various well-known classes of near-rings, such as strongly regular, equiprime, weakly commutative (right or left permutable) near-rings etc. It is also proved that these notions coincide with each other if N is a ring, but they are different if N is a near-ring. Some useful results are given.

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VI Timetable

	Wed 28	Thurs 29	Fri 30	Sat 31	Sun 1
9.00	Macintyre	Wood	Şengün	Zhou	Kirby
10.00	Moosa	Dries	Poonen	Guil	Süer
11.00	*coffee*				
11.30	Zilber	Günaydın	Kowalski	Lomp	Lewis
12.30	*lunch*				
14.30	Roquette	Zelevinsky		Muranov	
15.30	*parallel talks*			‖ *talks*	
16.30	*coffee*			*coffee*	
17.00	*parallel talks*			‖ *talks*	
18.00	*reception*				

Parallel talks, session 1

	Wed	Thurs	Sat
15.30	Mermut	Büyükaşık	Akbari
15.50	Nişancı	Kılıçarslan Cansu	Boukaroura
16.10	Baser	Demir	Ghaffarzadeh
17.00	Albaş	Ersoy	Gismatullin
17.20	Toksoy	Yılmaz	Abbaspour
17:40		Aygün	

Parallel talks, session 2

	Wed	Thurs	Sat
15.30	Halupczok	Onal	Tekcan
15.50	Malinin	Özcan	Bizim
16.10	Neman	Özdemir	Aksoy
17.00	Tolstykh	Sökmez	Türkmen
17.20	Alizade	Uğurlu	
17:40		Seven	