Antalya Cebir Günleri IX

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Brian Hartley'in anısına

Welcome to Antalya Algebra Days. This year's meeting, the ninth meeting in the series, is dedicated to the great mathematician and good friend Brian Hartley. Each of us has a reason to attend this year's conference. Some of us were Hartley's students; some, his friends; some know him from his papers. When I first sent out invitations to attend and contribute to this conference, everybody accepted within two days. This is a sign of the of the friendship Hartley has kept in our hearts after thirteen years without him. I thank everybody who has come, especially those who have travelled long distances. I understand and appreciate your support for this conference.

I also would like to thank heartily my colleagues and friends Ali Nesin, Alev Topuzoğlu and Sinan Sertöz for allowing me to invite more speakers in group theory this year.

I would like to thank the indispensible couple of organizers Ayşe Berkman and David Pierce: without their constant support my job would have been difficult. We are indebted to Otto Kegel, Alexandre Zalesskii and Victor Mazurov for their scientific guidence and help in the organization of this conference. They were with us whenever we needed help. Finally we are deeply grateful to all the scientific committee members for their support.

Our special thanks go to Tamer Koç and Şükran Demir of Tivrona Tours, who have met all of our requests smilingly.

I hope you enjoy the conference.

Mahmut Kuzucuoğlu

Contents

1 Hour talks

Black box groups and groups of finite Morley rank

Alexandre Borovik

My talk is a brief survey of strange and still mysterious connections between the theory of groups of finite Morley rank [3] and a chapter of computational group theory known as *black box recognition of finite groups* [1, 2, 6].

I will also briefly mention a new development: theory of asymptotic classes and measurable theories [4, 5] which suggest a new bridge between the theory of finite simple groups and model theory of groups.

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Noetherian Hopf algberas

Ken Brown

Following the discovery of quantum groups in the 1980s and the exploration of many Hopf algebras related to quantum groups in the years following, a very large menagerie of noetherian Hopf algebras had become available for study by the mid-1990s. I will review some of what is known about noetherian Hopf algebras, and explain some open questions about them.

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Necklaces and free Lie algebras

Roger Bryant

I shall first talk about the well-known formula (Witt's formula) that gives the number of primitive necklaces of length n and the dimension of the nth homogeneous component of a free Lie algebra. I shall then describe some recent results concerned with the module structure of a free Lie algebra under the action of a group. Let G be a group, K a field and V a finite-dimensional KG-module. Let $L(V)$ be the free Lie algebra over K that has V as a subspace and every basis of V as a free generating set. The action of each element of G on V extends to a Lie algebra automorphism of $L(V)$. Thus $L(V)$ becomes a KG-module, and each homogeneous component $L^n(V)$ is a KG-submodule called the nth Lie power of V. I shall describe joint work with Manfred Schocker that gives a general decomposition theorem for Lie powers in terms of modules denoted by B_n. I shall also report on recent work with Marianne Johnson that clarifies the structure of these modules B_n.

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On Brian Hartley problem on maximal subgroups in branch groups.

Rostislav Grigorchuk

I will speak about the problem posted by Brian Hartley on existence of maximal subgroups in certain branch self-similar groups.

The solution of the problem was found by Ekaterina Pervova for the main case, that interested Bryan Hartley (including the first example of a group of intermediate growth, and Gupta-Sidki p-groups). The topic was farther developed in 2002 in joint work of J.S.Wilson and speaker. Namely the result of Pervova was generalized to all groups which are abstractly commensurable with the 3-generated 2-group $\mathcal{G}$ of intermediate growth. Unexpected consequence of this (and the developed technique) is the subgroup separability of the group $\mathcal{G}$ (quite rare property in group theory) and the fact that any infinite finitely generated subgroup of $\mathcal{G}$ is abstractly commensurable to $\mathcal{G}$. The overview of all this results, some new observation and a list of open questions will be presented in the talk.

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Locally finite simple Moufang loops

J. I. Hall

A Moufang loop is a binary system close to a group but satifying only a weak form of the associative law. Doro and Glauberman [1, 2] observed that there is a direct connection between simple Moufang loops and simple groups with triality. Using this correspondence, Liebeck [5] proved that nonassociative finite simple Moufang loops arise from split octonian algebras over finite fields. The techniques used in the study of locally finite simple groups (say, [3, 4]) can then be used to prove that Liebeck's result remains true with "locally finite" in place of "finite."

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Locally linear simple groups

Otto Kegel

The group algebra FG and the group G are two quite different structures (although closely related). Questions like:

> Is there an element h not equal to 1 of an extension H of G such that the element $1 - h$ belongs to the twosided Ideal $(FH)I$ where I is a given ideal of FG?

mix the two structures and ask for a suitable larger group H (so that the ideals of FG are induced by the ideals of H. (Brian Hartley was involved in the first paper treating this type of question, Passman: The algebraic structure of group rings (1977), p. 387).

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Barely Transitive Groups

Mahmut Kuzucuoğlu

A group G has a barely transitive representation if G acts on an infinite set Ω transitively and faithfully and every orbit of every proper subgroup is finite. A group is called a barely transitive group if it has a barely transitive representation. One can show easily that a group G is barely transitive if and only if G has a subgroup H such that $|G : H|$ is infinite, $\bigcap_{x \in G} H^x = \{1\}$ and for every proper subgroup K of G, $|K : K \cap H| < \infty$: here H denotes the stabilizer of a point.

The concept of a barely transitive permutation group was introduced by Hartley in [1], in the context of research on Heineken–Mohamed groups. There, it was pointed out that, an infinite group is barely transitive in its regular permutation representation if all proper subgroups of that group are finite, and so the study of barely transitive groups can be thought of as a natural generalization of the research arising from the Schmidt problem. Besides the trivial examples of groups of type C_{p^∞}, there are examples of non-abelian imperfect locally finite barely transitive groups given by Hartley. On the other hand infinite quasi-finite groups in their right regular representations are barely transitive. In particular the p-groups constructed by Olshanskii are the examples of simple, periodic, finitely generated barely transitive groups.

The structure of imperfect barely transitive groups is fairly well understood. In fact they are locally finite. The situation with perfect barely transitive groups is quite different even in the class of locally finite groups. Does there exist a locally finite barely transitive group that coincide with its derived subgroup? This question, posed in [1] see also [3, Question 15.20] is still not answered. Basic properties of locally finite barely transitive groups are studied in [2] We will give a survey on barely transitive groups.

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Geometry of classical groups over rings

Alexander A. Lashkhi

Classical Groups can be defined when the scalars only form a ring (commutative in most cases). The methods used in their study when the scalars form a field can be slightly extended to local rings; but for more general rings, they don't apply any more, and new ideas were needed. They were brilliantly provided by O. T. O'Meara, J. Tits and F. Veldkamp. More recently, unexpected connection of Classical Groups with K-theory has been discovered (A. Hahn, D. James, U. Brehm, A. Bak and others) [1]–[5].

One of the main problems in this field of study is to translate the specific maps from the geometrical point of view (perspectivities, collineations, harmonic maps) into algebraic language (representation by the [semi]-linear isomorphisms) —the Fundamental Theorems of Geometric Algebra. For different geometries (affine, projective, symplectic, orthogonal, unitary) and different rings we will consider this and other similar problems.

A. Defining the perceptivity (in the classical way) it's essential that the geometry is the lattice with complements. This property is false for rings. Consequently, it's necessary to change the definition of perspectivities. Note that none single ring versions of the theorem about perspective maps is not known. With this goal should be important the definition: submodule $A \subseteq X$ is D-complement to the submodule $B \subseteq X$ if $A \cap B = 0$ and $\mathrm{PG}(K, A \oplus B) \cong \mathrm{PG}(K, X)$. Taking into consideration this one for the left Ore domains desirable results could be obtained.

B. For the projective line harmonic maps and existence of K. von Staudt's theorem was considered by various of authors. Positive answers were given for some specific rings. However for the (commutative) principal ideal domains the theorem of K. von Staudt is invalid. Defining the projective space as the set of all free one-dimensional submodules for some general noncommutative rings in condition for non-bijective harmonic maps is given and the complete analog of von Staudt's theorem is obtained.

C. For free modules over the rings with invariant basis property (IBN-ring) the collineations and lattice isomorphisms of affine spaces and affine geometries was studied and thus the fundamental Theorem of Affine Geometry was proved.

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Asymptotic representation theory of finite groups: Evolution of a representation theory for locally finite groups?

Felix Leinen

In the early 1990s, A. E. Zalesskii discovered a neat and natural way to generalize notions from the representation theory of finite groups to locally finite groups – see his survey [15] in the notes of the Istanbul Conference.

For a locally finite group G, the role of representations is taken by so-called *inductive systems*, that is, by families of sets Φ_F of irreducible representations of the finite subgroups F of G with the compatability condition that Φ_{F_1} be precisely the set of irreducible constituents of the restrictions to F_1 of the representations in Φ_{F_2} whenever $F_1 \leqslant F_2$. Clearly, inductive systems reflect the asymptotic behaviour of the representations of the finite subgroups of G. And A. E. Zalesskii proved that the ideals in the complex group algebra of G are in 1-1-correspondence with the inductive systems of complex representations of the finite subgroups of G.

Further, for our locally finite group G, the role of characters is taken by *normalized positive definite class functions* $\mathbb{C}G \longrightarrow \mathbb{C}$, which are also studied in the theory of $C^\star$-algebras. Due to a theorem by A. M. Vershik and S. V. Kerov [14], convexly indecomposable such functions (the "irreducible characters" of G) arise as direct limits of irreducible normalized characters of the finite subgroups of G. Every normalized positive definite class function gives rise to an ideal in $\mathbb{C}G$; the converse fails.

In principle, the above connections open the possibility to generalize classical representation theory to locally finite groups. But in practice, it is very hard to understand the immersion of the finite subgroups in a given locally finite group G and their effect on the branching rules for irreducible characters. At least, this information is available when the group G is built up from a well-understood family of finite subgroups in a systematic uniform way. This happens for example with many simple locally finite groups.

First results about positive definite class functions for simple locally finite groups were obtained by E. Thoma [13] and H.-L. Skudlarek [12] more than 30 years ago. Around that time, K. Bonvallet, B. Hartley, D. S. Passman, and M. K. Smith [1] observed that the complex group algebra of P. Hall's universal locally finite group *ULF* admits only the three unavoidable ideals.

In the early 1990s, A. E. Zalesskii [16] and B. Hartley [5, 4, 3] pursued the topic further. This was the starting point for a series of papers [11, 10, 9, 8, 7, 6] by O. Puglisi and F. Leinen with the aim to settle the situation completely for simple locally finite groups. Our work relies on S. Delcroix's distinction of simple locally finite groups into various classes [2]. At present, open questions remain for so-called *groups of 1-type* (these groups are built up from "tame" Kegel covers with alternating quotients), but also for finitary isometry groups over finite fields.

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Variants of the Cycle Class Map

James D. Lewis

The celebrated Hodge conjecture in its classical form, is a statement about the image of a particular cycle class map from an algebraic cycle group on a non-singular complex projective variety to singular cohomology. We give a precise statement of this conjecture from an algebraic point of view. We then discuss a variant of this conjecture, called the Hodge-Deligne conjecture. Next, and motivated in part by constructions in algebraic K-theory, we consider twisted variants of this cycle class map construction, and explain the advantages of this construction over the original cycle class map.

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Quotients of large Lie Groups and Lie Algebras

A. Yu. Olshanskii

We present recent results obtained jointly with D. I. Osin and Yu. A. Bahturin.

Recall that a group G is *large* if some finite index subgroup of G admits a surjective homomorphism onto a non-cyclic free group. Given a subset S of a group G, we denote by $\langle S \langle^G$ the normal closure of S in G. In the recent paper [1], Lackenby proved the following theorem.

Let G be a large group, H is a normal subgroup of G of finite index admitting a surjective homomorphism onto a non-cyclic free group, $g_1, \ldots, g_k$ a collection of elements of H. Then the quotient group $G/\langle g_1^n, \dots, g_k^n \rangle^G$ is large for all but finitely many $n \in \mathbb{N}$.

We provide a shorter proof of this theorem based just on the Schreier formula and Baumslag–Pride theorem of 1978 on the largeness of a group given by $n \geqslant 2$ generators and $n-2$ relations.

Thus our approach is very elementary. However it leads to new constructions. We obtain new examples of finitely generated infinite torsion groups. In particular, our examples enjoys in addition the following properties: the orders of elements are square free; every nilpotent section (i.e., a factor-group of a subgroup) is abelian; every section is residually finite.

Also an analog of the theorem and other results are obtained for restricted Lie algebras over fields of prime characteristic.

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Invariant ideals in abelian group algebras

Donald S. Passman

If H is a nonidentity group, then the group algebra $K[H]$ always has at least three distinct ideals, namely 0, the augmentation ideal $\omega K[H]$, and $K[H]$ itself. Thus it is natural to ask if groups exist for which the augmentation ideal is the unique nontrivial ideal. In such cases, we say that $\omega K[H]$ is *simple*. Certainly H must be a simple group for this to occur and, since the finite situation is easy enough to describe, we might as well assume that H is infinite simple. The first such examples, namely algebraically closed groups and universal groups, were offered in 1976. From this, it appeared that such groups would be quite exotic and rare. But A. E. Zalesskiĭ has shown that, for locally finite groups, this phenomenon is really the norm. Indeed, for all locally finite infinite simple groups, the characteristic 0 group algebras $K[H]$ tend to have very few ideals.

While some work still remains to be done on the simple group case, it nevertheless makes sense to move on to the second stage of this program. As suggested by B. Hartley and A. E. Zalesskiĭ, we should next consider certain abelian-by-(quasi-simple) groups. Specifically, these are the locally finite groups H having a minimal normal abelian subgroup V, with H/V infinite simple (or perhaps just close to being simple). Note that $G = H/V$ acts as automorphisms on V, and hence on the group algebra $K[V]$. Furthermore, if I is any nonzero ideal of $K[H]$, then it is easy to see that $I \cap K[V]$ is a nonzero G-stable ideal of $K[V]$. Thus, for the most part, this second stage is concerned with classifying the G-stable ideals of $K[V]$. Even in concrete cases, this turns out to be a surprisingly difficult task. Fortunately, there has been some progress on this problem, and the goal of this talk is to survey these results. For the most part, the methods used here are quite different from the usual group ring techniques.

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Thom polynomials and their positivity

Piotr Pragacz

The global behavior of singularities is governed by their *Thom polynomials* [16], [1], [2], [4], [5]. Knowing the Thom polynomial of a singularity, one can compute the cohomology class represented by the points of a map, having this singularity. For a more detailed account, we refer to [8] in the booklet of Antalya Algebra Days 2006.

Computation of the Thom polynomial of a singularity is a difficult task, even for "easiest" singularities. We are especially interested in the computation of the Thom polynomials containing as a parameter the difference of dimensions between the target and source of a map [15].

We widely use the *Schur function* techniques [6].

The talk has two goals:

(i) New, explicit computations of Thom polynomials for the singularities $I_{2,2}$ and A_i via their Schur function expansions [9], [10], [11], [12], [7]. These computations use the "method of restriction equations" of Rimanyi et al. for computing Thom polynomials [15], [2].

(ii) Investigation of the structure of the Thom polynomials expanded in the Schur function basis. Combining the approach to Thom polynomials via classifying spaces of singularities [5] with the Fulton-Lazarsfeld theory of cone classes and positive polynomials for ample vector bundles [3], we show – with Andrzej Weber – that the coefficients of the Schur function expansions of the Thom polynomials of nontrivial stable singularities are nonnegative with positive sum [13]. Then we present a generalization of this result to invariant cones in representations [14].

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From FLT to Finite Groups. The remarkable career of Otto Grün.

Peter Roquette

Students who start to learn group theory will soon be confronted with the theorems of Grün (or some generalization). Immediately after publication in the mid 1930s [1] these theorems found their way into group theory textbooks [2], and they turned out to be of fundamental importance in connection with the classical Sylow theorems. But little is known about the mathematician whose name is connected with those theorems.

In my talk I shall report on the remarkable mathematical career of Otto Grün. The talk will be based largely on the letters which were exchanged between Otto Grün and Helmut Hasse. I found 50 such letters in the archives of the University of Göttingen, from 1932 to 1972. They document the fascinating story of a mathematician, quite rare in our time, who was completely self-educated, without having attended university, and nevertheless succeeded, starting at age 44, to give important contributions to mathematics, in particular to group theory. Grün's first papers are concerned with class field theory around Fermat's Last Theorem (FLT). It was Hasse who pointed out to him the group theoretical problems in this connection, which then turned Grün's main interest to group theory.

We shall mention several documents about Grün which were found after publication of our paper [3].

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Coxeter Length

Peter Rowley

The notion of the length of an element in a Coxeter group frequently plays an important role in understanding various aspects of the Coxeter group. Indeed, there are many results concerning Coxeter groups which are proved by induction on the length of elements. In this talk we introduce a length function on subsets of a Coxeter group (which generalizes the length function on elements) and investigate some of its properties.

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Commutator maps, measure preservation and character methods

Aner Shalev

Given a group G we study the commutator map $f: G \times G \to G$ where $f(x,y) = x^{-1}y^{-1}xy$. When G is finite and has very few representations in some sense, we show that the map f is almost measure preserving. In particular almost all elements of such groups are commutators.

The main applications of this technique are for finite simple groups. We prove results towards a longstanding conjecture of Ore (which is still open), and confirm a conjecture of Guralnick and Pak on the Product Replacement Algorithm. This is joint work with Shelly Garion.

I will also mention recent results with Liebeck and O'Brien on Ore's conjecture, and with Larsen on more general word maps and Waring type problems.

Our methods combine representation theory with probabilistic arguments. In particular the so called Witten zeta function encoding character degrees plays a key role in the proofs.

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Centralizers in Locally Finite Groups

Pavel Shumyatsky

It is well-known that centralizers play fundamental role in the theory of locally finite groups. The main theme of our talk will be the following question.

Let G be a locally finite group admitting an automorphism ϕ of finite order such that the centralizer $C_G(\phi)$ satisfies certain finiteness conditions. What impact does this have on the structure of the group G?

Sometimes the impact is quite strong and we will give a survey of results illustrating this phenomena. In particular, we concentrate on results where G is shown to have a large nilpotent or soluble subgroup.

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Countably recognizable classes of groups

Howard Smith

A subgroup-closed class C of groups is countably recognizable if, whenever a group G has all of its countable subgroups in C, then G too belongs to C. (For a class C of subgroups that is not subgroup-closed there is an appropriately amended version of the definition of countable recognizability.) We will trace briefly the development of a theory of countable recognizability, highlighting several of the main results along the way, before presenting some new results on classes related to (local) solubility/nilpotency and finiteness of MalâĂŹcev rank.

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Modules with Ascending Chain Condition on Submodules with a Bounded Number of Generators

Patrick Smith

Given a positive integer n, a module satisfies n-acc provided every ascending chain condition of n-generated submodules terminates. This notion seems to have arisen first in the theory of Abelian groups and the Baumslag brothers generalized it to more general groups and to other algebraic systems. We shall consider it in the context of modules. A typical theorem is that if R is a commutative Noetherian one-dimensional domain then every direct sum of cyclic R-modules satisfies n-acc for every positive integer n. On the other hand if R is a (not necessarily commutative) right and left Noetherian ring then every direct product of copies of the right R-module R satisfies n-acc for every positive integer n.

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Asymptotically good codes with transitive automorphism groups

Henning Stichtenoth

One of the main open problems in algebraic coding theory is the question whether the class of cyclic codes is asymptotically good. This means: Is there a sequence of cyclic codes $(C_i)_{i\geqslant 0}$ over the finite field $\mathbb{F}_q$ whose parameters $[n_i, k_i, d_i]$ (length, dimension, minimum distance) satisfy

$$n_i \to \infty, \quad \liminf_{i\to\infty}(k_i/n_i) > 0 \quad \text{and} \quad \liminf_{i\to\infty}(d_i/n_i) > 0.$$

Cyclic codes $C \subseteq \mathbb{F}_q^n$ have the property that their automorphism group is a transitive subgroup of the symmetric group S_n. Hence it is natural to consider the class of *transitive* codes. Our main result is:

Theorem 1 *The class of codes over $\mathbb{F}_q$ (where q is a square) with transitive automorphism group is asymptotically good.*

The proof of the theorem uses Goppa's algebraic-geometric (AG) codes over function fields. The main ingredient is a new asymptotically good tower of function fields $F_0 \subseteq F_1 \subseteq F_2 \subseteq \dots$ over $\mathbb{F}_q$ with the property that all extensions $\mathbb{F}_n/\mathbb{F}_0$ are galois. With a careful choice of the divisors that define the corresponding AG codes one then obtains a transitive action of the Galois group on the codes.

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On torsion in free central extensions of groups

Ralph Stöhr

Let G be a group given by a free presentation $G = F/R$ and consider the quotient

$$F/[\gamma_c R, F] \qquad (*)$$

where $\gamma_c R$ denotes the c-th term of the lower central series of R and $c \geqslant 2$. This quotient is a free central extension of $F/\gamma_c R$, which is in its turn an extension of $G = F/R$ with free nilpotent kernel $R/\gamma_c R$. While the quotient $F/\gamma_c R$ is always torsion-free, elements of finite order may occur in $\gamma_c R/[\gamma_c R, F]$, the centre of $(*)$. In the case where $c = 2$ and $R = F'$, that is for the free centre-by-metabelian group $F/[F'', F]$, this was discovered by C.K. Gupta who proved in 1973 that the free centre-by metabelian group of rank $n \geqslant 4$ contains an elementary abelian 2-group of rank $\binom{n}{4}$ in its centre. This was a major surprise at the time as it was the first example of a relatively free group that is given by a single multilinear commutator identity and is not torsion-free. In a pioneering paper of 1977 Yu.V. Kuz'min identified Gupta's torsion subgroup with the fourth homology group of the free abelian group F/F' reduced modulo 2, thus linking the problem to homology of groups. In the eighties there was a series of papers by Yu.V. Kuz'min focusing on the case $c = 2$, and there was a series of papers by the speaker, including a joint paper [1] with the late Brian Hartley, on the general case $c \geqslant 2$. One of the main achievements of all these papers was a precise identification of the torsion subgroup $\mathrm{t}(\gamma_c R/[\gamma_c R, F])$ of $(*)$ in the case when c is a prime, say $c = p$, in terms of homology. Namely, if G has no elements of order p, then

$$\mathrm{t}(\gamma_p R/[\gamma_p R, F]) \cong H_4(G, \mathbb{Z}_p)$$

where $\mathbb{Z}_p = \mathbb{Z}/p\mathbb{Z}$ regarded as a trivial G-module. In the early nineties there was a single addition to this body of results for the case $c = 4$: for G without elements of order 2 there is an isomorphism $\mathrm{t}(\gamma_4 R/[\gamma_4 R, F]) \cong H_6(G, \mathbb{Z}_2)$. This was the last word on the subject for a long time. The methods used in the eighties and nineties appeared to be exhausted. Very recently, new results by Bryant, Erdmann and Schocker on modular Lie representations made it possible to make further progress on the torsion subgroup of $(*)$. In joint work with Marianne Johnson we have been able to identify this torsion subgroup for $c = 6$ (for G without elements of order 2 and 3). The result was a big surprise to us. In my talk I will review the history of the problem, and then I will focus on these recent developments.

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Simple Locally Finite Groups Of Diagonal Type

Simon Thomas

It is generally accepted that it is impossible to classify all countable simple locally finite groups, at least in the conventional sense of parametrizing the isomorphism types in some reasonable way. Instead, according to Hartley [5], we should look for well-behaved classes that can be classified, and try to increase the scope of these as much as possible. For example, the simple locally finite linear groups were explicitly classified in the early 1980s by several authors [1, 2, 4, 6]; and a little later, the simple locally finite groups of finitary linear transformations were explicitly classified by Hall [3]. In this talk, I will consider the question of whether a satisfactory classification exists for the class of simple locally finite groups of *diagonal type*; i.e. those simple locally finite groups which can be expressed diagonal limits of products of finite alternating groups.

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Lie algebras of infinite differential operators

Efim Zelmanov

I will discuss some new examples of nil algebras of finite Gelfand–Kirillov dimension and their possible connections with

(i) old problems,

(ii) Grigorchuk groups,

(iii) Physics.

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2 Half-hour talks

Triply factorized groups and nearrings

Bernhard Amberg

In the theory of factorized groups $G = AB$ with subgroups A and B one often has to deal with groups of the following form $G = AB = AM = BM$, where the normal subgroup M of G satisfies the intersection property $A \cap M = A \cap M = 1$. Such triply factorized groups—even with nonabelian M—may be constructed by using (local) nearrings.

A nearring R is an algebraic structure with two operations " $+$ " and " $\cdot$ " such that R is a not necessary abelian group under the operation " $+$ " and R is a semigroup under the operation " $\cdot$ " so that a one-sided distributive law holds. A nearring R with a unit element 1 is called local if the set of all invertible elements forms a subgroup of the additive group of R.

If L is the subgroup of all non-invertible elements of the local nearring R, then the set $1 + L$ is a subgroup of the multiplicative group of R acting on L, such that the semidirect product of $(1 + L)$ with L is a triply factorized group G, where M is isomorphic to L and A and B are both isomorphic to $1 + L$.

Conversely, it can be shown that for every triply factorized group G with $A \cap B = 1$ there exists a sub-nearring of the nearring of all mappings from M into itself, such that the given triply factorized group can be obtained via this construction.

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Subgroups of finitary permutation groups

A. O. Asar

This work continues the study of the subgroups of totally imprimitive groups of finitary permutations which was started in [1]. Some characterizations of non FC-subgroups and subnormal subgroups of stabilizers of these groups are obtained, and sufficient conditions are given for these groups to have proper subgroups having an infinite orbit.

Let G be a totally imprimitive subgroup of FSym(Ω), where Ω is infinite. In this work we continue the study of the following problem which was started in [1]: Can G contain a proper non FC-subgroup such that every orbit of every proper subgroup of it is finite or does every subgroup of G has an infinite orbit? It is well known that if every orbit of every proper subgroup of G is finite, then G is a minimal non FC-group (see [12, Theorem 1] or [3, Lemma 8.3D]). In [1], some characterizations of G were obtained based on the properties of point stabilizers. Also it was shown that if G satisfies the normalizer condition, then it is a p-group and G' is a minimal non FC-group. (The converse of this result is also true.) Furthermore sufficient conditions were given for G to contain a proper subgroup having an infinite orbit.

In the present work the following properties of subgroups of G are obtained. The intersection of two distinct conjugates of a subgroup of G is always an FC-group and in particular, G is a minimal non FC-group if and only if any two proper subgroups of G generate a proper subgroup. A subnormal subgroup of a stabilizer which is either solvable or of finite exponent is contained in a normal subgroup of finite exponent. (This together with [1, Lemma 2.5] gives a complete characterization of the subnormal subgroups of a stabilizer). Now suppose that G is a p-group. Then the kernel of a permutation representation with respect to a non-trivial block is always non-trivial. Finally sufficient conditions are given for G, when it is a p-group, to contain a proper subgroup having an infinite orbit. Also it is shown that these conditions are satisfied by Wiegold's example given in [12]. In fact it is easy to see that this example and its derived subgroup contain many proper subgroups having infinite orbits.

In view of the important reduction theorems given in [2] and [6] a complete determination of the types of the subgroups of G will settle the following well known problem: Does there exist a perfect locally finite minimal non FC-(p-group)?

For the definition and the important properties of totally imprimitive groups the reader is referred to [3, 8, 9] and [10]. Before stating the main results of this work we need to make a definition. Let $G \leqslant Sym(\Omega)$. For any non-trivial block Δ for G let

$$S_G(\Delta) = \{x \in G : supp(x) \subseteq \Delta\}.$$

It is easy to see that $S_G(\Delta)$ is a normal subgroup of $G_{\{\Delta\}}$ (see [1, Lemma 2.3(a)]). If $S_G(\Delta)$ is not contained in $G_{\{\Pi\}}$ for any block $\Pi \subset \Delta$, then Δ will be called a **dominant block** for G. Note that if $S_G(\Delta)$ is transitive on Δ then Δ is dominant. Let

$$\Delta_1 \subset \Delta_2 \subset \cdots \subset \Delta_n \subset \dots$$

be an ascending chain of non-trivial blocks for G whose union is Ω. If each Δ_i is dominant, then this chain is called a **dominant chain** for G. We also let $Ker(\Delta)$ denote the kernel of the natural permutation representation of G with respect to Δ (i.e, into $\mathrm{FSym}(\Sigma)$, where $\Sigma = \{x(\Delta) : x \in G\}$).

The main results of this work are stated below.

Theorem 1 *Let G be a totally imprimitive subgroup of* $\mathrm{FSym}(\Omega)$*, where Ω is infinite. Then the following hold:*

(i) a Let W be a subgroup of G and let $g \in G \setminus N_G(W)$. Then $W \cap W^g$ is an FC-group. Thus if W is not an FC-group and if $W \leqslant Y \leqslant G$, then $N_G(W) \leqslant N_G(Y)$;

(ii) G is a minimal non FC-group if and only if any two proper subgroups of G generate a proper subgroup of G.

Theorem 2 *Let G be a totally imprimitive subgroup of* $\mathrm{FSym}(\Omega)$*, where Ω is infinite. Let Δ be proper block for G and let S be a subnormal subgroup of $G_{\{\Delta\}}$. Then $\langle S^x : x \in G\rangle$ has finite exponent if one of the following holds:*

(i) S is solvable;

(ii) S has finite exponent.

Theorem 2 (i) generalizes [1, Theorem 1.1]. If G is a minimal non FC-group, then subnormality is not needed when $\exp(S)$ is finite by [1, Remark 1.10]. But subnormality is needed when S is only solvable since a point stabilizer always contains an abelian subgroup of infinite exponent. Note also that if G is a minimal non FC-group then it is a p-group, and so $\langle S^x : x \in G\rangle$ is nilpotent of finite exponent by [8, Theorem 2.4(ii)] or [3, Lemma 8.3D(ii)].

Theorem 3 *Let G be a totally imprimitive p-subgroup of* $\mathrm{FSym}(\Omega)$*, where Ω is infinite. Let Δ be a finite block for G and let M be the kernel of the representation of G with respect to Δ. Then $\mathrm{Core}_G(N_G(G_{\{\Delta\}}) \neq M$. In particular if Δ is non-trivial then $\mathrm{Core}_G(G_{\{\Delta\}}) \neq 1$.*

(For more on the kernels the reader is referred to [7].)

Theorem 4 *Let G be a totally imprimitive p-subgroup of* $\mathrm{FSym}(\Omega)$*, where Ω is infinite. Suppose that G has a non-trivial block Δ such that any non-trivial block for G containing Δ is dominant. Then G contains a proper subgroup that has an infinite orbit.*

Corollary 1 *Let G be a totally imprimitive p-subgroup of* $\mathrm{FSym}(\Omega)$*, where Ω is infinite. Suppose that G has an ascending chain of non-trivial blocks*

$$\Delta = \Delta_o \subset \Delta_1 \subset \cdots \subset \Delta_n \subset \ldots$$

whose union is equal to Ω such that for each $n \geqslant 0$ the following holds:

- $G_{\{\Delta_{n+1}\}} = N_G(G_{\{\Delta_n\}})$ *and*
- *there exists a cycle $y_{n+1} \in G_{\{\Delta_{n+1}\}}$ such that $G_{\{\Delta_{n+1}\}} = \langle y_{n+1} \rangle G_{\{\Delta_n\}}$.*

Then G contains a proper subgroup that has an infinite orbit.

It can be shown that the hypotheses of Theorem 4 Corollary 1 are satisfied by the example given in [12].

The notation and the definitions are standard and may be found in [3] and [11] (see also [1]).

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Quasi-permutation representations of finite groups

Houshang Behravesh

If G is a finite linear group of degree n, that is, a finite group of automorphisms of an n-dimensional complex vector space (or, equivalently , a finite group of non-singular matrices of order n with complex coefficients), we shall say that G is a quasi-permutation group if the trace of every element of G is a non-negative rational integer. The reason for this terminology is that, if G is a permutation group of degree n, its elements, considered as acting on the elements of a basis of an n-dimensional complex vector space V, induce automorphisms of V forming a group isomorphic to G. The trace of the automorphism corresponding to an element x of G is equal to the number of letters left fixed by x, and so is a non-negative integer. Thus, a permutation group of degree n has a representation as a quasi-permutation group of degree n. See [2].

By a quasi-permutation matrix we mean a square matrix over the complex field $\mathbb{C}$ with non-negative integral trace. Thus every permutation matrix over $\mathbb{C}$ is a quasi-permutation matrix. For a given finite group G, let $p(G)$ denote the minimal degree of a faithful permutation representation of G (or of a faithful representation of G by permutation matrices), let $q(G)$ denote the minimal degree of a faithful representation of G by quasi-permutation matrices over the rational field $\mathbb{Q}$, and let $c(G)$ be the minimal degree of a faithful representation of G by complex quasi-permutation matrices. See [1].

By a rational valued character we mean a character χ corresponding to a complex representation of G such that $\chi(g) \in \mathbb{Q}$ for all $g \in G$. As the values of the character of a complex representation are algebraic numbers, a rational valued character is in fact integer valued. A quasi-permutation representation of G is then simply a complex representation of G whose character values are rational and non-negative. The module of such a representation will be called a quasi-permutation module. We will call a homomorphism from G to $GL(n, \mathbb{Q})$ a rational representation of G and its corresponding character will be called a rational character of G. It is easy to see that

$$c(G) \leqslant q(G) \leqslant p(G)$$

where G is a finite group.

We will give algorithms for calculating $p(G)$, $q(G)$ and $c(G)$ where G is a finite group and also when G has a unique minimal normal subgroup. Also we will give the old and new results on the quasi-permutation representation of finite groups. Finally we will state a question from Brian Hartley.

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Complete Semigroups of Binary Relations

Ya. I. Diasamidze

This is joint work with Sh. I. Makharadze.

Let X be an arbitrary nonempty set. D be complete $X-$semilattice of unions, i.e. it is some nonempty set of subsets from X which is closed with respect to the set-theoretic union of elements from D, and f be arbitrary mapping of the set X in the set D. To each mapping f we assign a binary relation α_f on the set X which satisfies conditions

$$\alpha_f = \bigcup_{x \in X} (\{x\} \times f(x)).$$

We denoted by $B_X(D)$ the set of all such α_f $(f : X \longrightarrow D)$. It is well-known that $B_X(D)$ is subsemigroup of the semigroup B_X which is called a complete semigroup of binary relations, defined by an $X-$semilattice of unions D.

The report is dedicated to semigroups $B_X(D)$. The issue on substantiation of researched semigroups and some of their properties is studied. Some early well-known classes of semigroups that proved to be complete semigroups of binary relations are pointed out. Divisibility conditions of binary relations are found out. Describtion of idempotent and regular elements; right units; exterior and irreducible elements; maximal subgroup of semigroups $B_X(D)$ is given. Some of the properties of finite $X-$semilattice unions are studied.

Further, semigroups $B_X(D)$ that are defined by semilattices, generated by sets of nonchainwise paires; $X-$chaines; elementary semilattices; nodal and generalized-elementary semilattices are studied.

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On fixed point free action

Gülin Ercan

Let A be a finite group acting on a (solvable) group G fixed point freely. A long standing conjecture is that then the Fitting height of G is bounded by the longest chain of subgroups of A. Even though this conjecture is known to hold for large classes of groups A when $(|G|, |A|) = 1$, it is still not much known without this condition. In this presentation the above conjecture without the coprimeness condition will be discussed.

This is joint work with İsmail Ş. Güloğlu.

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QF-schemes and quadratic forms over local rings

V. M. Levchuk

The results of [1, 5] receive a development. The solution of the construction's task of a normal view for quadratic forms over a local principal ideal ring $R = 2R$ with a QF-scheme of order 2 is shown. Recall, that group $G = R^*/R^{*2}$ together with its particular map of subsets from G refers to QF-scheme or the quadratic form scheme of ring R. There are QF-schemes with abstract group G of exponent 2 are considered. We give a combinatorial representation for number of projective congruence quadrics classes of the projective space over R with a nilpotent maximal ideal. For the projective planes the quadric's enumerations up to projective equivalence is given; we consider also the projective planes over rings with a non-principal maximal ideal. Some problems for QF-schemes are opened.

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Large characteristic subgroups and their applications to almost regular automorphisms

Natalia Makarenko

I present our recent joint results with E. I. Khukhro, which enable us to solve certain important problems concerning groups with almost regular automorphisms.

In the theory of groups with almost regular automorphisms the final aim is often to prove the existence of a nilpotent subgroup with bounds for the nilpotency class and index. But in many situations we can construct such a nilpotent subgroup, say K, of bounded class c and bounded index n only in some normal subgroup, say H, and not in the group G itself. To use induction we need to consider the quotient G/K and therefore it is important to have K to be characteristic in H. Sometimes we can consider the automorphic closure $\prod_{\alpha\in \mathrm{Aut}\, H} K^{\alpha}$, which is a characteristic nilpotent subgroup of class $\leqslant cn$. However in an induction on the length of a certain subnormal series it is desirable not to increase the nilpotency class of the subgroup at each step. We show that if a group has a nilpotent subgroup of class c and index n, then there is a characteristic nilpotent subgroup of the same class c whose index is bounded in terms of n and c. Moreover, we obtain the analogous result for a subgroup satisfying an arbitrary multilinear commutator identity. We also prove similar theorems for a nilpotent normal subgroup (or an ideal in a Lie algebra) with a bound for the rank (dimension) of the quotient group (algebra).

We now give precise formulations. We say for short that a certain quantity is $(a, b, \ldots)$-bounded if it is bounded above by some function depending only on $a, b, \ldots$ Let x_1, $x_2, \ldots$ be group variables.

Definition 1 *Multilinear commutators of weight* 1 *in the variables* x_i *are the variables* x_i *themselves. By induction,* multilinear commutators of weight $w > 1$ in the variables x_i *are commutators of the form* $\varkappa = [\varkappa_1, \varkappa_2]$*, where* $\varkappa_1$ *and* $\varkappa_2$ *are multilinear commutators with disjoint sets of variables of weights* w_1 *and* w_2 *for* $w = w_1 + w_2$*.*

A group G satisfies a multilinear commutator identity $\varkappa = 1$ if and only if the commutator (verbal) subgroup $\varkappa(G)$ obtained by replacing all the variables in the commutator $\varkappa$ by the group G is trivial.

Theorem 1 *Let* $\varkappa$ *be a multilinear commutator of weight* w*. If a group* G *contains a subgroup* H *of finite index* $|G : H| = n$ *satisfying the identity* $\varkappa(H) = 1$*, then* G *also contains a characteristic subgroup* C *satisfying the identity* $\varkappa(C) = 1$ *whose index is finite and* (n, w)*-bounded.*

Such a result was so far known in folklore only for abelian subgroups, that is, for $\varkappa = [x_1, x_2]$.

We say that an ideal J of a Lie algebra L is *automorphically-invariant* if J is invariant under all automorphisms of L. A Lie algebra L is *nilpotent of class* $\leqslant c$ if $\gamma_{c+1}(L) := [\underbrace{L, \ldots, L}_{c+1}] = 0$.

Theorem 2 *If a Lie algebra* L *over any field has a nilpotent ideal of nilpotency class* c *and of finite codimension* r*, then* L *has also an automorphically-invariant* *ideal of finite (r, c)-bounded codimension that is nilpotent of class $\leqslant c$.*

This result, as well as ideas in its proof, are used to prove a similar result for groups, where the role of dimension is played by rank.

Recall that a group has rank r if every finitely generated subgroup of it can be generated by r elements.

Theorem 3 *Suppose that a group G has a nilpotent normal subgroup H of nilpotency class c such that the quotient group G/H has finite rank r. If either*

(i) H is periodic, or

(ii) H is torsion-free,

then G has also a characteristic nilpotent subgroup C of nilpotency class $\leqslant c$ with quotient group G/C of finite (r, c)-bounded rank.

We apply the above theorems for obtaining some results on almost regular automorphisms, which had previously stalled precisely because of the need for converting subnormal series into normal. Theorem 1 proved to be useful in the study of almost regular automorphisms of order 4.

Theorem 4 *There exist a constant c and a function of a positive integer argument $f(m)$ such that if a finite group G admits an automorphism of order 4 with exactly m fixed points, then G has characteristic subgroups $N \leqslant H \leqslant G$ such that $|G/H| \leqslant f(m)$, the quotient group H/N is nilpotent of class $\leqslant 2$, and the subgroup N is nilpotent of class $\leqslant c$.*

This result gives an affirmative answer to Shumyatsky's question 11.126 in the "Kourovka Notebook".

Theorem 3 is applied to automorphisms of prime order that are almost regular in the sense of small rank of the fixed-point subgroup. In [1, Theorem 2] it was proved that if a finite nilpotent group G of derived length d admits an automorphism of prime order p with centralizer of rank r, then G has a subnormal series of (p, r, d)-bounded length whose smallest term is nilpotent of p-bounded class, while the other factors have (p, r, d)-bounded rank. Now this result can be strengthened. Using also the result in [2], where Hall–Higman type theorems were combined with powerful p-groups to obtain almost nilpotency of a finite group with such an automorphism, we now have the following corollary.

Corollary 1 *If a finite soluble group G of derived length d admits an automorphism of prime order p with centralizer of rank r, then G has characteristic subgroups $R \leqslant C \leqslant G$ such that the quotient group C/R is nilpotent of p-bounded class, the subgroup R has (p, r)-bounded rank, and the quotient G/C has (p, r, d)-bounded rank; if G is nilpotent, then one can put $R = 1$.*

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A construction of non-tame automorphisms of a free group of rank 3 in the variety $\mathbf{A}_p\mathbf{A}$

A. I. Papistas

Let F_3 be a free group of rank 3. For a non-negative integer m, $\mathbf{A}_m$ denotes the variety of all abelian groups of exponent of dividing m, interpreted in such a way that $\mathbf{A}_0 = \mathbf{A}$ is the variety of all abelian groups. Write $\mathbf{V}_m$ for the variety of all extensions of groups in $\mathbf{A}_m$ by groups in $\mathbf{A}$. For a prime integer p, let $M_3 = F_3(\mathbf{V}_p)$. The natural mapping from F_3 onto M_3 induces a homomorphism α from $\mathrm{Aut}(F_3)$ into $\mathrm{Aut}(M_3)$. An automorphism of M_3 which belongs to the image of α is called tame. We give a way of constructing non-tame automorphisms of M_3.

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The geometry of Zappa-Szep products of groups

Stephen J Pride

A group G is a *semidirect product* of its two subgroups H and K if:

(i) H is normal in G;

(ii) $H \cap K = 1$;

(iii) $HK = G$.

In such a situation we have a homomorphism

$$\psi : K \longrightarrow Aut(H),\ \psi_k(h) = khk^{-1}\ (h \in H, k \in K),$$

and then the multiplication is given by

$$(hk)(h'k') = h\psi_k(h')kk'.$$

Conversely, given a pair of groups H, K and a homomorphism $\psi : K \longrightarrow Aut(H)$, we can construct a group $\hat{G}$ consisting of the ordered pairs (h, k) with multiplication

$$(h, k)(h', k') = (h\psi_k(h'), kk').$$

Then $\hat{H} = \{(h, 1) : h \in H\}$, $\hat{K} = \{(1, k) : k \in K\}$ are isomorphic copies of H, K, and $\hat{G}$ is the semidirect product of $\hat{H}, \hat{K}$.

A *Zappa-Szep product* occurs when condition (1) above is not required to hold. To *construct* such a product from given groups H, K, two functions $K \times H \longrightarrow K$, $K \times H \longrightarrow H$ are needed with certain conditions. Most of the work on this construction has been algebraic (see [1, 2] and the references cited there). However, using techniques of geometric group theory one can get a different perspective on the construction. I will describe this in my talk, and use such techniques to obtain new results concerning these groups.

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Abelian normal subgroups of unipotent subgroups of Chevalley groups

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Throughout this paper $G(K)$ denotes a Chevalley group over an arbitrary field K, associated with the root system G. In this work the maximal abelian normal subgroups of the unipotent subgroup $UG(K)$ are described. The terminology from [2, 1, 4] will be used. The description was partially announced in [5]. For the classical types $G = A_n,\ B_n,\ C_n,\ D_n,\ {}^2A_m,\ {}^2D_{n+1}$ the results were obtained with V.M. Levchuk.

For example, the following theorem is proved:

Theorem 1 *Suppose that M is a maximal abelian normal subgroup of $UG(K)$.*

(i) *If $G = {}^2B_2$, then $M = Z_1\langle\alpha\rangle$ with an arbitrary $\alpha \notin Z_1$;*

(ii) *If $G = G_2$, or $3K = 0$, or $G = {}^2G_2$, then $M = Z_2 = U_2$;*

(iii) *If $G = G_2$ and $2K = 0$ and $|K| > 2$, then either $M = \{x_{a+b}(t)x_{2a+b}(td) \mid t \in K\}Z_2$ with an arbitrary $d \in K$ and $Z_2 = U_4$, or $M = U_3$;*

(iv) *If $G = G_2$ and $|K| = 2$, then $M = \langle x_a(1)x_{2a+b}(1)\rangle Z_2$, where $Z_2 = \langle x_{a+b}(1)x_{2a+b}(1)\rangle U_4$;*

(v) *If $G = G_2$, or $6K = K$, or $G = {}^3D_4$, then $M = Z_3 = U_3$;*

(vi) *If $G = {}^2F_4$, then $M = \{R_{3,-2}(t)R_{42}(td) \mid t \in K\}Z_4$ with an arbitrary $d \in K$, or $M = \langle R_{43}(a)\rangle R_{42}(K)Z_4$, where $a \in K^*$.*

The full description of the set $A_N(U)$ of large abelian normal subgroups of $U = UG(K)$ over the finite field $K = GF(q)$ [3, Problem (1.6)]) may be obtained from these results. For example, the next list of subgroups from $A_N(U)$ and their order $a(U)$ may be obtained from Theorem 1:

G	$A_N(U)$	$a(U)$	K
${}^2B_2(q)$	$Z_1\langle\alpha\rangle \quad (\alpha \notin Z_1)$	$2q$	
${}^2G_2(q)$	U_2	q^2	
${}^3D_4(q^3)$	U_3	q^5	
$G_2(q)$	U_3	q^3	$6K = K$
	U_2	q^4	$3K = 0$
	U_3, $\{x_{a+b}(t)x_{2a+b}(td) \mid t \in K\}U_4 \quad (\lvert K\rvert > 2)$	q^3	$2K = 0$
	$\langle x_a(1)x_{2a+b}(1),\ x_{a+b}(1)x_{2a+b}(1)\rangle U_4$ or $M = U_3$ or $M = X_{a+b}U_4$	q^4	$\lvert K\rvert = 2$
${}^2F_4(q)$	$\langle R_{43}(a)\rangle R_{42}(K)U_5$	$2q^5$	

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Vanishing Products of One Dimensional Classes in Group Cohomology

Ergün Yalçın

Let G be a 2-group. We are interested in the algebra structure of the cohomology of G in mod 2 coefficients, denoted usually by $H^*(G, \mathbb{Z}/2)$. Let σ_G denote the the product of all (nontrivial) one dimensional classes in $H^*(G, \mathbb{Z}/2)$. A classical theorem by Serre [1] states that $(\sigma_G)^2 = 0$ provided that G is not an elementary abelian 2-group. We recently improved this result in the following way:

Theorem 1 ([2]) *If G is a nonabelian 2-group, then $\sigma_G = 0$*

In the proof, we use a cohomological characterization of abelian p-groups. Namely, we show that a p-group G is nonabelian if and only if the inflation map

$$\inf_{G/\Phi(G)}^{G} : H^3(G/\Phi(G), \mathbb{Z}) \to H^3(G, \mathbb{Z})$$

has a nontrivial kernel (here $\Phi(G)$ denotes the Frattini subgroup of G). The key lemma which leads to this characterization is purely group theoretical.

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From proofs about finite groups to probabilistic algorithms for black box groups

Şükrü Yalçınkaya

The main purpose of this talk is to develop an analogy between the methods for recognizing a black box group and the classification of the finite simple groups. Following this talk, Alexandre Borovik will talk about the relations between black box groups and groups of finite Morley rank.

A *black box group* G is a finite group whose elements are encoded as 0-1 strings of uniform length and the group operations are performed by an oracle ('black box'). Given strings representing $g, h \in G$, the black box can compute the strings representing $g \cdot h$, g^{-1} and decide whether $g = h$. In problems naturally arising in computational group theory, some additional information is usually available, for example, the group can be given as a group of matrices over a finite field; another natural assumption is that we have an *order oracle* which determines the orders of elements.

In this context, a natural task is to find a probabilistic algorithm which determines the isomorphism type of a group within given (arbitrarily small) probability of error. Under various mild assumptions, there are positive answers to this question in class of finite simple groups; for example, one can determine the isomorphism type of a finite simple group of Lie type with an order oracle [2].

In this talk, we propose a uniform approach for recognizing simple groups of Lie type which follows the computational version of the classification of the finite simple groups. Similar to the inductive argument on centralizers of involutions which plays a crucial role in the classification project, our approach is based on a recursive construction of the centralizers of involutions in black box groups [4, 5].

We present an algorithm which constructs a long root $\mathrm{SL}_2(q)$-subgroup in a finite simple group of Lie type of odd characteristic. Following this construction, we take the Aschbacher's "Classical Involution Theorem" [1] as a model in the final recognition algorithm and we construct all root $\mathrm{SL}_2(q)$-subgroups corresponding to the nodes in the extended Dynkin diagram, that is, we construct the extended Curtis - Phan - Tits [6, 7, 8] presentation of the finite simple groups of Lie type of odd characteristic. In particular, we construct all subsystem subgroups which can be read from the extended Dynkin diagram. With this approach it is also possible to determine whether the p-core (or "unipotent radical") $O_p(G)$ of a black box group G is trivial or not where $G/O_p(G)$ is a finite simple group of Lie type of odd characteristic p answering a well-known question of Babai and Shalev [3].

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Hartley's theorem on representations of the general linear groups: a new approach

A. E. Zalesski

In 1986 Brian Hartley [1] obtained the following interesting result:

Theorem 1 *Let K be a field, E the standard $GL_n(K)$-module, and V be an irreducible finite-dimensional $GL_n(K)$-module over K with $\dim V > 1$. Let H be a finite subgroup of $GL_n(K)$. Suppose that H is not of exponent 2 and the restriction of E to H contains a free KH-module. Then V contains a free KH-submodule.*

His proof is based on deep properties of the duality between irreducible representations of the general linear group $GL_n(K)$ and the symmetric group S_n. We suggest another proof based on general principles of representation theory of algebraic groups, more precisely, on properties of the weight systems of algebraic group representations. Therefore, it seems to be more conceptual and in a sense more transparent. We also expect that our method will have further applications.

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3 Contributed talks

Extensions of weakly supplemented modules

Rafail Alizade

Throughout, R is a ring with identity and modules are unital left R-modules. If $N + K = M$ and $N \cap K \ll M$, then K is called a *weak supplement* of N (see, [2]). M is a *weakly supplemented* module if every submodule of M has a weak supplement. A submodule N of M is called *coclosed* in M if $N/K \ll M/K$ for some $K \subseteq M$ then $K = N$.

Theorem 1 *Let $0 \to L \to M \to N \to 0$ be a short exact sequence. If L and N are weakly supplemented and L has a weak supplement in M then M is weakly supplemented.*

If L is coclosed in M then the converse holds, that is if M is weakly supplemented then L and N are weakly supplemented.

The hypothesis that L has a weak supplement can not be omitted. To give an example we firstly prove the following lemmas.

Lemma 1 *Let R be a Dedekind domain and K be the field of quotients of R. Then ${}_RK$ is weakly supplemented.*

Lemma 2 *Let R be a Dedekind domain and $\{\mathfrak{p}_i\}_{i\in I}$ be an infinite collection of distinct maximal ideals of R. Let $M = \prod_{i\in I}(R/\mathfrak{p}_i)$ be the direct product of the simple R-modules $R/\mathfrak{p}_i$ and $T = T(M)$ be the torsion submodule of M. Then M/T is divisible, therefore $M/T \cong K^{(J)}$ for some index set J and $\operatorname{Rad} M = 0$.*

Example 1 *Let R and M be as in Lemma 2 and $T = \oplus_{i\in I}(R/\mathfrak{p}_i)$ be the torsion submodule of M. Note that T is semisimple, so it is weakly supplemented. Let N be a submodule of M such that $N/T \cong K$. Then N/T is weakly supplemented by Lemma 1. Note that $\operatorname{Rad} N = 0$ by Lemma 2 and N is not semisimple because*

$N/T \cong K$ is not semisimple. Hence by Corollary 2.3 in [1], N is not weakly supplemented.

Moreover we have proved that a Dedekind domain R is semilocal iff every direct product of simple R-modules is weakly supplemented.

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One particle N-representability

Murat Altunbulak

The talk will be a presentation of the joint-paper with Prof. Alexander Klyachko. The problem in question is about relation between spectra of a mixed state ρ of N fermion system and its one point reduced matrix $\rho^{(1)}$. We briefly review the history of the problem and give two different solutions, one in terms of linear inequalities on spectra of ρ and $\rho^{(1)}$ and another in terms of irreducible components of certain plethysms of the unitary group. We combine these solutions to produce all constraints on one point reduced matrix of a pure state for fermion systems up to rank 8. Some of them were conjectured by Borland and Dennis [1] more than 30 years ago.

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Characterizations of Minimal Non-Solvable Fitting p-Groups

Ahmet Arıkan

Let G be a minimal non-solvable Fitting p-group. We mainly prove that G has $2^{\aleph_0}$ subgroups and G has a homomorphic image $\overline{G}$ with a proper subgroup $\overline{X}$ such that $C_{\overline{G}}(\overline{X}) = 1$ which gives a generalization of Asar's result in [1]. We also define the solvabilizer of a solvable group and consider minimal non-solvability problem in another context.

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From abstract co-Galois theory to field theoretic co-Galois theory

Şerban A. Basarab

An abstract (topological and group theoretic) approach of the field theoretic co-Galois theory is initiated in the recent papers [1] and [3]. Given a profinite group Γ and a quasi-cyclic discrete group A on which Γ acts continuously, several classes of closed subgroups of Γ and related classes of subgroups of the cocycle group $Z^1(\Gamma, A)$, called *radical, hereditarily radical, Kneser, hereditarily Kneser* and *co-Galois*, are defined, investigated and characterized.

A more general frame for the abstract co-Galois theory is the object of a recent project of the author [4], [5]. In this more general setting one considers a continuous action of a profinite group Γ on a (not necessarily commutative) profinite group G together with a continuous 1-cocycle $\eta : \Gamma \longrightarrow G$ with the property that the profinite group G is topologically generated by the image $\eta(\Gamma)$. The main object of study is the natural Galois connection between the topological lattice of all closed subgroups of Γ and the topological lattice of all Γ-invariant closed normal subgroups of G. The theory developed in [1], [3], which we may call *cyclotomic abstract co-Galois theory* is the first stage of this more general theory, where $G := \mathrm{Hom}(Z^1(\Gamma, A), A)$ is the Pontryagin dual of the discrete Abelian torsion group $Z^1(\Gamma, A)$ with the action induced by the action of Γ on the quasi--cyclic group A, while the cocycle $\eta : \Gamma \longrightarrow G$ is defined by $\eta(\gamma)(\alpha) = \alpha(\gamma)$ for $\alpha \in Z^1(\Gamma, A), \gamma \in \Gamma$.

The main results of the field theoretic co-Galois theory are easily obtained from their cyclotomic abstract co-Galois theoretic versions. Moreover, new results in the field theoretic co-Galois theory are derived from specific results of the cyclotomic co-Galois theory [5]. On the other hand, the more general frame for the abstract co-Galois theory provides a suitable extension of the frame of the classical field theoretic co-Galois theory.

We illustrate the introductory remarks above with an extension of the abstract Kneser criterion [1, Theorem 1.20], an application of a result concerning the so called *quasi-co-Galois subgroups* to the description of the *minimal non-co-Galois hereditarily Kneser algebraic separable field extensions* and an example of *strongly co-Galois algebraic separable field extensions* related to the model theory of Henselian fields and p-adically closed fields.

Surjective cocycles and minimal non-Kneser structures

The next result is an extension of [1, Theorem 1.20].

Proposition 1 [4, Proposition 6.23] (Abstract Kneser Criterion) *Let G be a profinite Γ-operator group and $\eta \in Z^1(\Gamma, G)$ be a* generating cocycle, i.e. *the profinite group G is topologically generated by $\eta(\Gamma)$. The following assertions are equivalent for any Γ-invariant closed normal subgroup $\mathbf{a}$ of G.*

(1) *The induced cocycle $\eta_{\mathbf{a}} \in Z^1(\Gamma, G/\mathbf{a})$ is surjective.*

(2) $\mathbf{a} \not\subseteq \mathbf{m}$ *for all Γ-invariant open normal subgroups $\mathbf{m}$ of G wich are maximal with the property that the induced cocycle $\eta_{\mathbf{m}} \in Z^1(\Gamma, G/\mathbf{m})$ is not surjective.*

The following class of finite algebraic structures arises naturally from the abstract Kneser criterion above.

Definition 1 *A triple (Γ, G, η) consisting of a finite group Γ, a finite Γ-operator group G and a generating cocycle $\eta \in Z^1(\Gamma, G)$, with $\Delta := \mathrm{Ker}(\eta)$, is called a* minimal non-Kneser structure *if the following conditions are satisfied :*

(1) $\eta(\Gamma) \neq G$,

(2) *for every Γ-invariant normal subgroup $\mathbf{a} \neq \{1\}$, $(\Gamma : \eta^{-1}(\mathbf{a})) = (G : \mathbf{a})$, and*

(3) *the cocycle η is* normalised, i.e. $\bigcap_{\gamma \in \Gamma} \gamma \Delta \gamma^{-1} = \mathrm{Fix}_{\Gamma}(G) \cap \Delta = \{1\}$.

Problem 1 *Classify up to isomorphism the minimal non-Kneser structures.*

A proper subclass of minimal non-Kneser structures is provided by the next obvious lemma.

Lemma 1 [4, Lemma 7.3] *Let $\eta \in Z^1(\Gamma, G)$ be a normalised generating cocycle such that $\eta(\Gamma) \neq G$. Then (Γ, G, η) is a minimal non-Kneser structure whenever the Γ-operator group G is simple,* i.e. $\{1\}$ *and* $G \neq \{1\}$ *are its only Γ-invariant normal subgroups.*

The next results provide some classes of minimal non-Kneser structures including the very simple ones which occur in [1, Lemma 1.18 and Theorem 1.20].

Lemma 2 [4, Lemma 7.7] *Let (Γ, G, η) be a minimal non-Kneser structure. Assume that Γ and G are Abelian. The following assertions hold.*

(1) *G is an Abelian p-group for some prime number p, Γ acts faithfully on G and the generating cocycle η is injective. Let $p^n, n \geqslant 1$, be the exponent of G.*

(2) *There exists a unique minimal non-zero Γ-submodule of G.*

(3) *The image R of the canonical ring morphism $\mathbb{Z}/p^n\mathbb{Z}\,[\Gamma] \longrightarrow \mathrm{End}(G)$ is a finite commutative local ring of characteristic p^n, and Γ, identified with a subgroup of R^*, generates R as $\mathbb{Z}/p^n\mathbb{Z}$-module, in particular as ring.*

Proposition 2 [4, Proposition 7.8] *Let $\eta \in Z^1(\Gamma, G)$, where Γ and G are Abelian and finite, G is a p-group for some prime number p, while Γ is not a p-group. Then, the following assertions are equivalent.*

(1) *(Γ, G, η) is a minimal non-Kneser structure.*

(2) *The local ring $R \subseteq \mathrm{End}(G)$ is a finite field $k \cong \mathbb{F}_q, q = p^f, f \geqslant 1$ for $p \neq 2$, $f \geqslant 2$ for $p = 2$, G is a one-dimensional k-vector space identified with k^+, the group Γ, identified with a subgroup of the multiplicative group k^*, is cyclic of order $1 \neq r \mid (q-1)$ such that f is the order of $p \bmod r \in (\mathbb{Z}/r\mathbb{Z})^*$, and $\eta \in Z^1(\Gamma, k^+) = B^1(\Gamma, k^+) \cong (\mathbb{Z}/p\mathbb{Z})^f$ is, up to multiplication by elements in k^*, the coboundary $u \in \Gamma \mapsto u - 1 \in k^+$.*

Proposition 3 [4, Proposition 7.9] *Let $\eta \in Z^1(\Gamma, G)$, where Γ and G are finite Abelian p-groups for some prime number p. Assume that the finite local ring $R \subseteq \mathrm{End}(G)$ defined in Lemma 3 is principal. Then, the following assertions are equivalent.*

(1) *(Γ, G, η) is a minimal non-Kneser structure.*

(2) *One of the following conditions is satisfied:*

   (i) $n = 1$, i.e. *the characteristic of R is p, and $R \cong \mathbb{F}_p[x]/(x^m)$ with $m \geqslant 2$ and $(m, p) = 1$.*

   (ii) $p = 2, n = 2$, i.e. *the characteristic of R is 4, and $R \cong \mathbb{Z}/4\mathbb{Z}[x]/(f(x), 2x^e)$, where $f \in \mathbb{Z}/4\mathbb{Z}[x]$ is an Eisenstein polynomial of degree $e \geqslant 1$, in particular, the nilpotency index $m = 2e$ is even and R is free of rank e as $\mathbb{Z}/4\mathbb{Z}$-module.*

   (iii) $p = 2, n = 2$, i.e. *the characteristic of R is 4, and $R \cong \mathbb{Z}/4\mathbb{Z}[x]/(f(x), 2x^t)$, where $f \in \mathbb{Z}/4\mathbb{Z}[x]$ is an Eisenstein polynomial of degree $e \geqslant 2, 0 < t < e$, and the nilpotency index $m = e + t$ is odd.*

In all three cases above, the R-module G can be identified with $R^+, R/\mathfrak{m} \cong \mathbb{F}_p, \Gamma = 1 + \mathfrak{m}$, and the injective cocycle $\eta \in Z^1(\Gamma, R^+)$ is unique up to multiplication with elements from R^* and summation with homomorphisms from Γ to $H^0(\Gamma, R^+) \cong \mathbb{Z}/p\mathbb{Z}$.

The next result is an immediate consequence of Propositions 4 and 5.

Corollary 1 [4, Corollary 7.11] *Let* (Γ, G, η) *be a minimal non-Kneser structure. Assume that* G *is Abelian of exponent* e *and the action of* Γ *is induced by a character* $\chi : \Gamma \longrightarrow (\mathbb{Z}/e\mathbb{Z})^*$, *i.e.* $\gamma g = \chi(\gamma) g$ *for all* $\gamma \in \Gamma, g \in G$. *Then* $G \cong \mathbb{Z}/e\mathbb{Z}$, *where* e *is either an odd prime number* p *or* 4. *In the first case,* $(\Gamma, G, \eta) \cong (U, \mathbb{F}_p^+, u \mapsto u-1)$, *where* $U \cong \Gamma \cong \mathbb{Z}/r\mathbb{Z}$, $2 \leqslant r \mid (p-1)$, *is the unique subgroup of order* $r = |\Gamma|$ *of the multiplicative group* $\mathbb{F}_p^*$ *of the prime field* $\mathbb{F}_p$, *acting by multiplication. In the latter case,* $(\Gamma, G, \eta) \cong (\mathbb{Z}/2\mathbb{Z}, \mathbb{Z}/4\mathbb{Z}, 1 \bmod 2 \mapsto 1 \bmod 4)$ *with the unique nontrivial action of* $\mathbb{Z}/2\mathbb{Z}$ *on* $\mathbb{Z}/4\mathbb{Z}$.

As a consequence of Corollary 6, we find again [1, Lemma 1.18, Theorem 1.20].

From quasi-co-Galois subgroups to quasi-co-Galois field extensions

Let Γ be a profinite group acting continuously on a discrete quasicyclic group $A \subseteq \mathbb{Q}/\mathbb{Z}$. Put $\mathfrak{Z} := Z^1(\Gamma, A)^\vee = \mathrm{Hom}(Z^1(\Gamma, A), A)$. The Abelian profinite group $\mathfrak{Z}$ becomes a compact Γ-module with the action defined by $(\gamma\psi)(g) = \gamma(\psi(g))$ for $\gamma \in \Gamma, \psi \in \mathfrak{Z}, g \in Z^1(\Gamma, A)$, while the continuous map $\eta : \Gamma \longrightarrow \mathfrak{Z}, \gamma \mapsto \eta(\gamma) : g \mapsto g(\gamma)$ is a generating cocycle. We denote by $\mathbb{L}(G)$ the lattice of all subgroups of a subgroup $G \subseteq Z^1(\Gamma, A)$ and by $\mathbb{L}(\Gamma \mid \Delta)$ the lattice of all closed subgroups of Γ lying over a closed subgroup $\Delta \subseteq \Gamma$. The lattices $\mathbb{L}(Z^1(\Gamma, A))$ and $\mathbb{L}(\Gamma) := \mathbb{L}(\Gamma \mid \{1\})$ are related through the canonical reversing maps $G \mapsto G^\perp := \bigcap_{g \in G} \mathrm{Ker}(g), \Delta \mapsto \Delta^\perp := \{ g \in Z^1(\Gamma, A) \mid \Delta \subseteq \mathrm{Ker}(g) := g^\perp \}$.

First we recall some basic notions of the cyclotomic abstract co-Galois theory.

Definition 2 *A subgroup* $G \subseteq Z^1(\Gamma, A)$ *is* Kneser *if the induced cocycle* $\eta_G \in Z^1(\Gamma, \mathfrak{Z}/G^\vee)$ *is surjective, so the induced map* $\Gamma/G^\perp \longrightarrow \mathfrak{Z}/G^\vee$ *is a homeomorphism of profinite spaces.*

G is co-Galois *if G is Kneser and the lattices* $\mathbb{L}(G)$ *and* $\mathbb{L}(\Gamma \mid G^\perp)$ *are canonically anti-isomorphic.*

A closed subgroup $\Delta \subset \Gamma$ *is* Kneser *if there exists a Kneser subgroup* $G \subseteq Z^1(\Gamma, A)$ *such that* $\Delta = G^\perp = \mathrm{Ker}(\eta_G)$.

Δ *is* hereditarily Kneser *if each* $\Lambda \in \mathbb{L}(\Gamma \mid \Delta)$ *is Kneser.*

Δ *is* co-Galois *if there exists a co-Galois subgroup* $G \subseteq Z^1(\Gamma, A)$ *such that* $\Delta = G^\perp$. *[Such a co-Galois subgroup G is unique].*

Δ *is* quasi-co-Galois *if* Δ *is maximal with the property that it is hereditarily Kneser but not co-Galois.*

The quasi-co-Galois subgroups of Γ are exactly the open Kneser subgroups Δ which are not co-Galois, but each $\Lambda \in \mathbb{L}(\Gamma \mid \Delta) \setminus \{\Delta\}$ is co-Galois. A more precise description of the quasi-co-Galois subgroups is given by the next statement.

Proposition 4 [5, Lemma 3.1] *The quasi-co-Galois subgroups of* Γ *are exactly those open subgroups* $\Delta \subseteq \Gamma$ *which satisfy one of the following four conditions.*

(1) $\Delta \lhd \Gamma$, $\widehat{1/4} \in A^\Delta \setminus A^\Gamma$, *and* $\Gamma/\Delta \cong (\epsilon_4^\perp/\Delta) \rtimes (\Gamma/\epsilon_4^\perp) \cong \mathbb{D}_8$.

(2) $\Delta \lhd \Gamma$, $\widehat{1/8} \in A^{\Delta}$, $\widehat{1/4} \notin A^{\Gamma}$, *and the factor group* $\Gamma/\Delta \cong Q$ (the group of quaternions) *has the presentation*

$$\Gamma/\Delta \cong \langle\, \sigma, \tau \,|\, \sigma^4 = 1, \sigma^2 = \tau^2, \sigma\tau\sigma^{-1} = \tau^{-1}\,\rangle = \langle\, \sigma, \tau \,|\, \sigma\tau = \tau^{-1}\sigma,\ \tau\sigma = \sigma^{-1}\tau\,\rangle,$$

with the action on $(1/8)\mathbb{Z}/\mathbb{Z} \subseteq A^{\Delta}$ *given by* $\sigma\,\widehat{1/8} = \widehat{3/8}$ *and* $\tau\,\widehat{1/8} = \widehat{1/8}$.

(3) $\widehat{1/8} \in A \setminus A^{\Delta}, \widehat{1/4} \in A^{\Delta} \setminus A^{\Gamma}, \Delta \cap \epsilon_8^{\perp} \lhd \Gamma$, *and the factor group* $\Gamma/(\Delta \cap \epsilon_8^{\perp})$ *has the presentation*

$$\Gamma/(\Delta \cap \epsilon_8^{\perp}) \cong (\epsilon_4^{\perp}/(\Delta \cap \epsilon_8^{\perp})) \rtimes (\Gamma/\epsilon_4^{\perp}) \cong$$

$$\langle\, \sigma, \tau, \delta \,|\, \sigma^2 = \tau^4 = \delta^2 = (\sigma\tau)^2 = [\sigma, \tau\delta] = [\tau, \delta] = 1\,\rangle,$$

with the action on $(1/8)\mathbb{Z}/\mathbb{Z}$ *given by* $\sigma\,\widehat{1/8} = -\widehat{1/8}, \tau\widehat{1/8} = \widehat{1/8}$ *and* $\delta\,\widehat{1/8} = \widehat{5/8}$.

(4) $\Delta \lhd \Gamma$, *there exist an odd prime number* p *and a divisor* $r > 1$ *of* $p - 1$ *such that* $\widehat{1/pr} \in A^{\Delta}$, $\widehat{1/p} \notin A^{\Gamma}$, $\widehat{1/l} \in A^{\Gamma}$ *for all* $l \in \mathcal{P}$ *with* $l \,|\, r$, *and*

$$\Gamma/\Delta \cong (\epsilon_p^{\perp}/\Delta) \rtimes (\Gamma/\epsilon_p^{\perp}),\ \Gamma/\epsilon_p^{\perp} \cong \mathbb{Z}/r\mathbb{Z},\ \epsilon_p^{\perp}/\Delta \cong \mathbb{Z}/p\mathbb{Z}.$$

[For $n \in \mathbb{N}$, ϵ_n denotes the coboundary $\gamma \mapsto (\gamma - 1)\widehat{1/n}$, while $\mathcal{P}$ stands for the set of all prime natural numbers where 2 is replaced by 4.]

The field theoretic notions which are required for stating the field theoretic analogue of Proposition 7 above are briefly recalled in the following.

Let E/F be an algebraic field extension. We denote by $T(E/F)$ the multiplicative group of all elements $x \in E^*$ which are radicals over F, *i.e.* $x^n \in F^*$ for some $n \in \mathbb{N} \setminus \{\,0\,\}$. The *co-Galois group* $\mathrm{Cog}(E/F)$ of the field extension E/F is the factor group $T(E/F)/F^*$, *i.e.* the torsion subgroup of the factor group E^*/F^*. By Hilbert's Satz 90, $\mathrm{Cog}(E/F)$ is canonically isomorphic with $Z^1(\Gamma, \mu)$ whenever E/F is Galois with Galois group Γ and μ is the multiplicative quasi-cyclic group of all roots of unity contained in E. A dictionary relating the group theoretic notions and the the corresponding field theoretic ones is transparent.

Definition 3 *A finite separable field extension* E/F *is* Kneser *if* $E = F(G)$ *for some subgroup* $G \subseteq T(E/F)$ *lying over* F^* *such that* $[E : F] = (G : F^*)$. *If, in addition, the lattice* $\mathbb{L}(E/F)$ *of all intermediate fields between* F *and* E *and the lattice* $\mathbb{L}(G \,|\, F^*)$ *are canonically isomorphic, we say that* E/F *is* co-Galois. *[In the latter case, the group* G *of radical elements is uniquely determined.]*

An algebraic separable field extension E/F *is* hereditarily Kneser *if each finite subextension* L/F *is Kneser.* E/F *is* co-Galois *if each finite subextension* L/F *is co-Galois.*

An algebraic separable field extension E/F *is called* quasi-co-Galois *if it is minimal with the property that it is hereditarily Kneser but not co-Galois.*

It turns out that the quasi-co-Galois field extensions are exactly those separable finite field extensions E/F which are Kneser, not co-Galois, but each subextension $L/F, L \neq E$ is co-Galois. As a consequence of Proposition 7, we obtain a precise description of the quasi-co-Galois field extensions as follows.

Proposition 5 [5] *Let E/F be an algebraic separable field extension. Then, the necessary and sufficient condition for E/F to be quasi-co-Galois is that one of the following situations hold :*

(1) $\mathrm{char}(F) \neq 2, \zeta_4 \notin F$, *and* $E = F(\zeta_4, \theta)$ *for some* $\theta \in E$ *satisfying* $\theta^4 = a$, *where* $a \in F \setminus \pm F^2$.

(2) $\mathrm{char}(F) \neq 2, 2 \in (-F^2) \setminus F^2$, *and* $E = F(\theta)$ *for some* $\theta \in E$ *such that* $a := \theta^8 \in (-F^2) \setminus (-F^4)$.

(3) $\mathrm{char}(F) \neq 2, 2 \notin \pm F^2, \zeta_4 \notin F$, *and* $E = F(\theta)$ *for some* $\theta \in E$ *such that* $a := \theta^8 \in (-F^2) \setminus (-F^4 \cup -4F^4)$.

(4) *there exist a prime number* $p \neq 2$ *and* $1 < r \mid (p-1)$ *such that the characteristic exponent of* F *is prime to* $pr, \zeta_p \in E \setminus F, \zeta_r \in F(\zeta_p), \zeta_l \in F$ *for all* $l \in \mathcal{P}, l \,|\, r, [F(\zeta_p) : F] = r$, *and* $E = F(\zeta_p, \theta)$ *for some* $\theta \in E$ *satisfying* $\theta^p = a$, *where* $0 \neq a \in F \setminus F^p$.

[For $n \in \mathbb{N} \setminus \{ 0 \}$, ζ_n denotes a primitive nth root of unity.]

From strongly co-Galois subgroups to strongly co-Galois field extensions

Proposition 6 [5, Lemma 2.8] *Let* Δ *be a* radical subgroup *of* Γ, i.e. $\Delta = \Delta^{\perp\perp}$. *Then the following statements are equivalent*

(1) $\Delta^{\perp}$ *is a co-Galois subgroup of* $Z^1(\Gamma, A)$.

(2) $\Delta^{\perp}$ *is a Kneser subgroup of* $Z^1(\Gamma, A)$.

(3) $A^{\Gamma}[p] = A^{\Delta}[p]$ *for all* $p \in \mathcal{P}$.

Call such a subgroup $\Delta \subseteq \Gamma$ strongly co-Galois

The field theoretic analogue of Proposition 9 due to Greither and Harrison [7] reads as follows.

Theorem 1 *The following statements are equivalent for a separable radical extension E/F.*

(1) E/F *is* $T(E/F)$*-Kneser.*

(2) E/F *is* $T(E/F)$*-co-Galois.*

(3) $\mu_p(E) \subset F$ *for all* $p \in \mathcal{P}$.

Call such an extension E/F strongly co-Galois.

The following class of strongly co-Galois extensions plays a key role in the model theory of Henselian fields and p-adically closed fields.

Theorem 2 [8, Theorem 3.8], [2, Proposition 1.1] *Let (F, v) be a Henselian valued field of characteristic zero. An algebraic field extension E of F is strongly co-Galois whenever E is a core-dense extension of F.*

In particular, if F is a p-valued Henselian field, E/F is strongly co-Galois whenever E is of the same p-rank as F.

Note that the radical structure theorem above is a main ingredient in the proof of an isomorphism theorem for algebraic extensions [8, Corollary 3.11], [6], and of a (relative) quantifier elimination theorem [8, Theorem 5.6], [2, Theorems A, B].

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Ratliff-Rush Monomial Ideals

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Let R be a Noetherian ring and let an ideal I in it be regular, that is, let I contain a nonzerodivisor. Then the ideals $(I^{l+1} : I^l)$, $l \geqslant 1$, increase with l. The union $\tilde{I} = \bigcup_{l \geqslant 1}^{\infty} (I^{l+1} : I^l)$ was first studied by Ratliff and Rush in [1]. They show that $(\tilde{I})^l = I^l$ for sufficiently large l and that $\tilde{I}$ is the largest ideal with this property. Hence, $\tilde{\tilde{I}} = \tilde{I}$. Moreover, they show that $\widetilde{I^l} = I^l$ for sufficiently large l. We call $\tilde{I}$ the Ratliff-Rush ideal associated to I, and an ideal such that $\tilde{I} = I$ a Ratliff-Rush ideal. The Ratliff-Rush reduction number of I is defined as $r(I) = \min \{l \in \mathbb{Z}_{\geqslant 0} \,|\, \tilde{I} = (I^{l+1} : I^l)\}$. The operation $\tilde{}$ cannot be considered as a closure operation in the usual sense, since $J \subseteq I$ does not generally imply $\tilde{J} \subseteq \tilde{I}$. An example from [2] shows this: let $J = \langle y^4, xy^3, x^3y, x^4\rangle \subset I = \langle y^3, x^3\rangle \subset k[x, y]$, then I is Ratliff-Rush but $x^2y^2 \in \tilde{J}\backslash\tilde{I}$.

One of the reasons to study Ratliff-Rush ideals is the following. Let I be a regular $\mathfrak{m}$-primary ideal in a local ring $(R, \mathfrak{m}, k)$. The Hilbert function $H_I(l) = \dim_k(R/I^l)$ is a polynomial $P_I(l)$ called the Hilbert polynomial of I for all large l. Then $\tilde{I}$ can be defined as the unique largest ideal containing I and having the same Hilbert polynomial as I.

Ratliff-Rush ideals associated to monomial ideals are monomial by definition, which makes the computations easier. There is always a positive integer L such that $\tilde{I} = I^{L+1} : I^L$, but it is not clear how big that L is (see Example 1.8 in [2]). If I is a monomial ideal and m is some monomial, then for all $l \geqslant 0$ we have

$$(mI)^{l+1} : (mI)^l = (m^{l+1}I^{l+1}) : (m^lI^l) = m(I^{l+1} : I^l). \qquad (\dagger)$$

Principal ideals are trivially Ratliff-Rush. Any non-principal monomial ideal J in the rings $k[x, y]$ and $k[[x, y]]$ can be written as $J = mI$, where m is a monomial and I is an $\langle x, y\rangle$-primary ideal; hence it suffices to consider $\langle x, y\rangle$-primary monomial ideals. Moreover, (†) shows that the Ratliff-Rush reduction numbers of I and mI are the same.

In this talk we show how to compute the Ratliff-Rush ideal associated to a monomial ideal in a certain class in the rings $k[x, y]$ and $k[[x, y]]$ and find an upper bound for the Ratliff-Rash reduction number for such an ideal. We start by giving some results about numerical semigroups that are crucial for our later work. We conclude by duscussing several useful examples.

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On the Cycle Structure of Permutation Polynomials of Finite Fields

Ayça Çeşmelioğlu

This is joint work with Wilfried Meidl and Alev Topuzoğlu. Permutation polynomials over $\mathbb{F}_q$ of degree $\leqslant q-1$ form a group under composition and subsequent reduction modulo $x^q - x$ which is isomorphic to S_q. Leonard Carlitz observed in his 1953 article [1] that any transposition $(0\ a)$, $a \in \mathbb{F}_q$ can be represented by the polynomial

$$P_a(x) = -a^2(((x-a)^{q-2} + a^{-1})^{q-2} - a)^{q-2}.$$

His result shows that the group of permutation polynomials over $\mathbb{F}_q$ is generated by x^{q-2} and $ax + b$, $a, b \in \mathbb{F}_q$, $a \neq 0$. The above mentioned isomorphism motivates the study of the cycle structure of permutation polynomials of $\mathbb{F}_q$. See [3], [4] for work on cycle structure of some specific polynomials of interest.

In this work we study the cycle structure of the polynomials

$$\mathcal{P}_n(x) = (((a_0x + a_1)^{q-2} + a_2)^{q-2} + \ldots + a_n)^{q-2} + a_{n+1}$$

over $\mathbb{F}_q$ with $a_0 \neq 0$ and note that any permutation polynomial of $\mathbb{F}_q$ can be expressed in this form for some $n \geqslant 0$, with $\mathcal{P}_0(x) = a_0x + a_1$. We define a rational function corresponding to $\mathcal{P}_n(x)$ and by using the known structure of $\mathcal{P}_1(x)$ or the cycle structure of rational linear transformations we determine the cycle structure of $\mathcal{P}_2(x)$ and $\mathcal{P}_3(x)$ completely . With the help of [2] we also give constructions for $P_n(x)$ with full cycle for arbitrary $n \geqslant 1$.

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Locally solvable° groups of finite Morley rank

Adrien Deloro

This is joint work with Eric Jaligot. The main conjecture concerning groups of finite Morley rank is the Cherlin–Zilber one, stating that any simple infinite group of finite Morley rank is isomorphic to an algebraic group over an algebraically closed field. A proof of (some reasonable part of) this conjecture is expected to run parallel to the classification of finite simple groups, and vice-versa.

As there is no Feit–Thompson theorem for groups of finite Morley rank, one has to assume there are involutions. Now as in the finite case, it is natural to consider at some point "locally solvable groups". This amounts to transfering results obtained on so-called "minimal connected simple groups" to a larger class. *Locally* here does not mean finitely generated, but comes from the following.

Definition 1 *Let G be a group. A subgroup $H \leqslant G$ is said to be* local *if there is some non-trivial abelian subgroup $A \leqslant G$ such that $H \leqslant N_G(A)$.*

Definition 2 *Let G be a group of finite Morley rank. G is said to be* locally solvable *if for any non-trivial abelian subgroup $A \leqslant G$, $N_G(A)$ is solvable.*

As people usually work with connected components (denoted by a $^\circ$), our definition is the following.

Definition 3 *Let G be a group of finite Morley rank. G is said to be* locally solvable$^\circ$ *if for any non-trivial abelian subgroup $A \leqslant G$, $N_G^\circ(A)$ is solvable.*

The only non-solvable locally solvable$^\circ$ algebraic group is $\mathrm{PSL}_2(\mathrm{K})$, which does have involutions.

Our main tools rely on a notion of unipotence in characteristic 0 developed by Jeffrey Burdges in his PhD [Bur04]. Namely one has to generalize the notion of a characteristic in the following way.

Definition 4 *A* unipotence parameter *is a pair $\tilde{q} = (p, d)$, where p is either prime or ∞, $d \in \mathbb{N} \cup \{\infty\}$, such that $p \neq \infty \Leftrightarrow d = \infty$.*

So if p is a real prime number, this is just indicating the characteristic, but if $p = \infty$ the notion of unipotence becomes graded (d is called the unipotence degree). In the latter case Burdges defined $\tilde{q}$-unipotent subgroups, and $\tilde{q}$-Sylow subgroups. As far as results and not definitions are concerned, both these notions provide good analogs of the prime characteristic ones.

There is an important uniqueness lemma that was first proved in the minimal connected simple framework [Del07a].

Lemma 1 (Uniqueness Lemma) *Let G be a locally solvable$^\circ$ group if finite Morley rank. Let $\tilde{q} = (p, d)$ be some unipotence parameter with $d > 0$, and let U be some $\tilde{q}$-Sylow subgroup of G.*

Assume U contains some $\tilde{q}$-unipotent subgroup U_1, and that U_1 contains some subset X such that the p-unipotence degree of $C^\circ(X)$ is not bigger than d.

Then U is the only $\tilde{q}$-Sylow subgroup of G containing U_1.

This lemma is strong enough to provide a good intersection control, allowing us to prove the following.

Theorem 1 (Mixed type theorem) *Let G be a locally solvable$^\circ$ group of finite Morley rank. Assume G has mixed type. Then G° is solvable*

Theorem 2 (Even type theorem) *Let G be a locally solvable$^\circ$ group of finite Morley rank. Assume G has even type. Then either G° is solvable or $G \simeq \mathrm{PSL}_2(\mathrm{K})$ for some algebraically closed field K of characteristic 2.*

Our results in odd type are not as positive yet.

Theorem 3 (Odd type algebraic theorem) *Let G be a locally solvable$^\circ$ group of finite Morley rank. Assume G has odd type and Pruefer-rank 1. Assume also that there is a toral involution i such that $C^\circ(i)$ is not a Borel subgroup. Then either G° is solvable or $G \simeq \mathrm{PSL}_2(\mathrm{K})$ for some algebraically closed field K of characteristic not 2.*

Theorem 4 (Odd type non-algebraic theorem) *Let G be a non-solvable locally solvable$^\circ$ group of finite Morley rank. Assume G has odd type and Pruefer--rank $\geqslant 2$. Then G has Pruefer-rank equal to 2, its Sylow subgroup is connected, and involutions are conjugate. Furthermore for any involution i of G, $C^\circ(i)$ is a Borel subgroup.*

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Algebras with skew-symmetric identity of degree 3

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Algebras with one of the following identities are considered:

$$[[t_1,t_2],t_3]+[[t_2,t_3],t_1]+[[t_3,t_1],t_2]=0, \text{(Lie-Admissible)}$$

$$[t_1,t_2]t_3+[t_2,t_3]t_1+[t_3,t_1]t_2=0,\ \text{(0-Lie-Admissible (shortly 0-Alia))}$$

$$\{[t_1,t_2],t_3\}+\{[t_2,t_3],t_1\}+\{[t_3,t_1],t_2\}=0, \text{(1-Lie-admissible (shortly 1-Alia))}$$

where $[t_1,t_2]=t_1t_2-t_2t_1$ and $\{t_1,t_2\}=t_1t_2+t_2t_1$. For algebra $A=(A,\circ)$ with multiplication $\circ$ denote by $A^{(q)}$ an algebra with vector space A and multiplication $a\circ_q b=a\circ b+q\,b\circ a$.

Theorem 1 *Any algebra with a skew-symmetric identity of degree 3 is (anti-) isomorphic to one of the following algebras:*

- *Lie-admissible algebra*
- 0-*Alia algebra*
- 1-*Alia algebra*
- *algebra of the form $A^{(q)}$ for some left-Alia algebra A and $q\in K$, such that $q^2\neq 0,1$.*

Example 1 $(\mathbf{C}[x],\circ)$ *under multiplication $a\circ b=\partial(a)\partial^2(b)$ is 1-Alia and simple.*

Example 2 $(\mathbf{C}[x], \star)$, *where* $a \star b = \partial^3(a)b + 4\partial^2(a)\partial(b) + 5\partial(a)\partial^2(b) + 2a\,\partial^3(b)$, *is* 0-*Alia and simple.*

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The Extension Class of a Subset Complex

Aslı Güçlükan

This is a joint work with E. Yalçın. Let G be a finite group and X be a finite G-set. The subset complex $\Delta(X)$ of X is defined as the simplicial complex whose simplices are subsets of X. The oriented complex of $\Delta(X)$ in mod 2 coefficients gives the following $\mathbb{F}_2G$-module extension

$$0 \to \mathbb{F}_2 \to C_{n-1}(\Delta(X)) \to \cdots \to C_0(\Delta(X)) \to \mathbb{F}_2 \to 0.$$

Let $\bar{\zeta}_X \in \mathrm{Ext}^n_{\mathbb{F}_2G}(\mathbb{F}_2, \mathbb{F}_2)$ denote the extension class for this extension. In the case $X = G/1$, the class $\zeta_{G/1}$ is an essential class which makes it interesting for group cohomology. This class is a mod 2 reduction of a class ζ_X which is introduced by Reiner and Webb [1]. They raised the question that for which finite groups $\zeta_{G/1} \neq 0$? We answer their question under mod 2 reduction:

Theorem 1 *Let G be a finite group. Then $\bar{\zeta}_{G/1}$ is non-zero if and only if G is an abelian 2-group isomorphic to $(\mathbb{Z}/2)^n \times (\mathbb{Z}/4)^m$ for some $n \geqslant 0$ and $m = 0, 1$.*

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Standard tableaux and Klyachko's Theorem on Lie representations

Marianne Johnson

Let $n \in \mathbb{N}$ and let $\lambda = (\lambda_1, \dots, \lambda_k)$ be a partition of n. That is, $\lambda_1 \geqslant \cdots \geqslant \lambda_k > 0$ and $\lambda_1 + \cdots + \lambda_k = n$. The Young diagram corresponding to λ is a collection of n boxes arranged in left justified rows with λ_i boxes in the ith row. A standard tableau of shape λ is a numbering of the Young diagram of λ with the numbers from $\{1, \dots, n\}$ in such a way that the entries strictly increase along every row and down every column. We say that an entry i is a descent in a standard tableau T if the entry $i+1$ occurs in any row below the row in which i appears. The sum of all descents in T is called the major index of T.

For example, $\lambda = (5, 3, 2, 1)$ is a partition of 11. Shown below is a standard tableau T of shape λ. The descents of T are $2, 6, 9$ and 10. Hence, the major index of T is $2 + 6 + 9 + 10 = 27$.

1	2	5	6	9
3	4	10		
7	8			
11				

We show that for all partitions λ of $n \geqslant 3$ ($\lambda \neq (2^2), (2^3), (1^n), (n)$) there exists a standard tableau of shape λ with major index coprime to n [1]. By a result of Kraśkiewicz and Weyman [3], this provides a new purely combinatorial proof of Klyachko's Theorem [2] on Lie representations of the general linear group.

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Modules of uniform Jordan type

Semra Öztürk Kaptanoğlu

Let G be an abelian p-group, M be a finitely generated $k[G]$-module, where k is a field of characteristic $p > 0$. We say M is of uniform Jordan type if the restriction of M to the subalgebra $k[\langle 1+x \rangle]$ has the same Jordan type as the restriction of M to the subalgebra $k[C_{p^r}]$ for some cyclic subgroup C_{p^r} of G for all x in the Jacobson radical of $k[G]$ with $\langle 1+x \rangle \cong C_{p^r}$.

We prove that the class of modules of uniform Jordan type contains endotrivial modules and it is closed under taking direct sums and tensor products.

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Connections between pseudo-free groups and RSA

Bilal Khan

Rivest introduced the notion of a pseudo-free group and showed that if Z_n is pseudo-free then it also satisfies several cryptographic assumptions, e.g. the Strong RSA Assumption and the difficulty of computing discrete logarithms. Later, Daniele Micciancio showed that the Strong RSA Assumption implies that Z_n is pseudo-free, thereby establishing an equivalence. In this talk, we will give a survey of cryptographic assumptions and their connections to the property of pseudo-freeness. We will then present new results on non-cyclic groups that are also pseudo-free.

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Finite Covers and Congruences

Elisabetta Pastori

This talk can be placed within the area of infinite permutation groups and their relations with model theory. For brief surveys about this area see for example [Ca] and [EMI].

After giving the definition of finite cover of a structure W (we give it in terms of the automorphism groups of the structures involved), we show a theorem which describes all the possible kernels of finite covers of a transitive structure W having all binding groups and fibre groups isomorphic to a simple non abelian regular group G, in terms of the $\mathrm{Aut}(W)$-congruences on W. This result has its basis in the article [EH] . Then we take in exam the case of $W = \Omega^{(n)}$, the set of ordered n-tuples with all distinct entries from an infinite set Ω, with $\mathrm{Aut}(W) = \mathrm{Sym}(\Omega)$ and we show a result of biinterpretability.

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Modules whose Maximal Submodules are Supplements

Dilek Pusat-Yılmaz

(Joint work with Engin Büyükaşık)

Let R be a ring and M be a unital left R-module. M is called an $MS-module$ if every maximal submodule of M is a supplement in M, and M is called an $MD-module$ if every maximal submodule of M is a direct summand of M.

In this talk some properties and some characterizations of MS-modules will be given.

Proposition 1 *Let $M = \sum_{i \in I} M_i$ for some index set I. Suppose M_i is an MS-module for every $i \in I$. Then M is an MS-module.*

Theorem 1 *Let R be a noetherian ring and M an R-module. M is an MS-module if and only if $M/D(M)$ has no maximal submodules where $D(M) = \sum\{Rm : m \in \Lambda\}$ and*

$\Lambda = \{m \in M : Rm \cap P \ll P \; \forall P \leqslant M \text{ such that } Rm \nsubseteq P\}$.

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Generalized PC-Groups And Generalized CC-Groups Of Matrices

Francesco Russo

Let $\mathfrak{X}$ be a class of groups. An element x of a group G is said to be $\mathfrak{X}C$-*central*, or an XC-*element,* if $G/C_G(x^G)$ satisfies $\mathfrak{X}$, where the symbol x^G denotes the normal closure of the subgroup $< x >$ in G (see [10]). A $\mathfrak{X}C$-central group G is also called *a group with* $\mathfrak{X}$*-conjugacy classes.* An XC-element x of a group G is also called *an element which has* $\mathfrak{X}$*-conjugacy classes.* The factor group $G/C_G(x^G)$ is a group of automorphisms of x^G and an old question of Group Theory is the knowledge of a group, starting from information on its group of automorphisms.

Following [10], a class $\mathfrak{X}$ of groups is said to be $\mathbf{S}_n$-*closed* if every normal subgroup of an $\mathfrak{X}$-group is an $\mathfrak{X}$-group. A class $\mathfrak{X}$ of groups is said to be $\mathbf{N}$-*closed* if the subgroup generated by any system of normal $\mathfrak{X}$-subgroups of a group is an $\mathfrak{X}$-group. A class $\mathfrak{X}$ of groups is said to be $\mathbf{P}$-*closed* if every extension of an $\mathfrak{X}$-group is an $\mathfrak{X}$-group. From now the symbol $\mathfrak{X}$ will denote a class of groups which is $\mathbf{S}_n$, $\mathbf{N}$ and $\mathbf{P}$-closed.

It is possible to introduce some series which generalize the usual notion of upper central series of a group (see for instance [10, Chapter 2]) thanks to the following remark.

Let G be a group. If x and y are XC-elements of G, then both $G/C_G(x^G)$ and $G/C_G(y^G)$ satisfies $\mathfrak{X}$, so $G/(C_G(x^G) \cap C_G(y^G))$ satisfies $\mathfrak{X}$. But the intersection of $C_G(x^G)$ with $C_G(y^G)$ lies in $C_G((xy^{-1})^G)$, so that $G/C_G((xy^{-1})^G)$ satisfies $\mathfrak{X}$ and xy^{-1} is an XC-element of G. Hence the XC-elements of G form a subgroup $X(G)$ and it is easy to see that $X(G)$ is characteristic in G.

This remark has been first noted by R. Baer for the class of FC-groups (see [10, Lemma 4.31]) and allows us to define the series

$$1 = X_0 \triangleleft X_1 \triangleleft \ldots \triangleleft X_\alpha \triangleleft X_{\alpha+1} \triangleleft \ldots,$$

where $X_1 = X(G)$, $X_{\alpha+1}/X_\alpha = X(G/X_\alpha)$ and

$$X_\lambda = \bigcup_{\alpha<\lambda} X_\alpha,$$

with α ordinal and λ limit ordinal. This series is a characteristic ascending series of G and it is called *upper XC-central series* of G. The last term of the upper XC-central series of G is called *XC-hypercenter* of G. If $G = X_\beta$, for some ordinal β, we say that G is an *XC-hypercentral* group of type at most β.

The *XC-length* of an XC-hypercentral group is defined to be the least ordinal β such that $G = X_\beta$, in particular when $G = X_c$ for some positive integer c, we say that G is *XC-nilpotent* of length c. A group G is said to be an *XC-group* if all its elements are XC-elements, that is, if G has XC-length at most 1. The first term $X(G)$ of the upper XC-central series of G is called *XC-center* of G and the α-th term of the upper XC-central series of G is called *XC-center of length α* of G. Roughly speaking, the upper XC-central series of G measures the distance of G to be an XC-group.

If $\mathfrak{X}$ is the class $\mathfrak{F}$ of finite groups, we have the notion of FC-hypercentral group and classical results can be found in [10, Chapter 4]. In this circumstance, we specialize the previous symbol X in the symbol F, for introducing XC-hypercentral series and related definitions. If $\mathfrak{X}$ is the class $\mathfrak{PF}$ of polycyclic-by-finite groups, we have the notion of PC-hypercentral group and a complete description of such groups can be found in [2]. In this circumstance, we specialize the previous symbol X in the symbol P, for introducing PC-hypercentral series and related definitions. If $\mathfrak{X}$ is the class $\mathfrak{C}$ of Chernikov groups, we have the notion of CC-hypercentral group and a complete description of such groups can be found in [3]. In this circumstance, we specialize the previous symbol X in the symbol C, for introducing CC-hypercentral series and related definitions.

Our approach to generalized central series of a group has been introduced by D.H.McLain, obtaining classical results in Theory of Generalized FC-groups (see [10, Theorem 4.37, Theorem 4.38]). Also [4] obtain similar results with this approach. One of the most recent improvement of McLain Theorems has been furnished by [1, Theorem A, Theorem B, Theorem C] for PC-hypercentral groups and CC-hypercentral groups.

On the other hand hypercentral groups, FC-hypercentral groups, PC- hypercentral groups and CC-hypercentral groups can be much different between themselves as it is shown either by the consideration of the infinite dihedral group or by means of examples in [1] and [5].

Our results are devoted to groups of matrices of PC-hypercentral groups and CC-hypercentral groups. A group G is called *linear of degree n over an arbitrary field K*, or a group of $n \times n$ matrices over an arbitrary field K, or briefly *linear,* if G is a subgroup of the group $GL(n, K)$, where n is a positive integer. Classical literature on linear groups is represented by [6], [9], [12], [13]. More recent is [8], where [8, Chapter 5] gives a survey on chains conditions in linear groups. These chains conditions have been first introduced in Russian literature (see [6], [9], [12]). We prove the following results, which extend [7, Theorem 2].

Theorem 1 *Let G be a linear group of degree n over an arbitrary field K. Then the following conditions are equivalent*

- *G is PC-nilpotent;*
- *G is PC-hypercentral;*
- *G is nilpotent-by-polycyclic-by-finite.*

Theorem 2 *Let G be a linear group of degree n over an arbitrary field K. If $\operatorname{char} K = 0$ and $G/Z(G)$ does not contain subgroups of infinite exponent, then the following conditions are equivalent*

- *G is FC-nilpotent;*
- *G is FC-hypercentral;*
- *G is nilpotent-by-finite;*
- *G is CC-nilpotent;*
- *G is CC-hypercentral.*

The construction in [11] will allow us to give examples of groups which satisfy Theorem A and Theorem B.

Key Words: Conjugacy classes; linear PC-groups; linear CC-groups; PC-hypercentral series; CC-hypercentral series.

MSC 2000: 20C07; 20D10; 20F24.

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Groups Elementary Equivalent to a Free 2-nilpotent Group of Finite Rank

Mahmood Sohrabi

In this talk I describe our joint work with Alexei Miasnikov on characterization of groups elementary equivalent to a free nilpotent group of arbitrary finite rank and class two. I'll outline a proof that a characterization similar to the one Belegradek, O. V. [1] found for groups elementary equivalent to the group of 3×3 unitriangular matrices over the rings of integers, which is a free 2-nilpotent group of rank 2, can be given for groups elementary equivalent to a free 2-nilpotent group of finite rank.

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The elementary theory of Thompson's group F is undecidable

Vladimir Tolstykh
(joint work with Valery Bardakov)

M. Sapir [3] asked whether the elementary theory of Thompson's group F is decidable (see [1] for the background information on Thompson's groups.) Since the group F is riddled with copies of the restricted wreath product $\mathbf{Z} \operatorname{wr} \mathbf{Z}$, he asked later whether there is a first-order definable (with parameters) copy of $\mathbf{Z} \operatorname{wr} \mathbf{Z}$ in F. Note that the elementary theory of the group $\mathbf{Z} \operatorname{wr} \mathbf{Z}$ is hereditarily undecidable, for the group $\mathbf{Z} \operatorname{wr} \mathbf{Z}$ first-order interprets the ring of integers $\mathbf{Z}$ [2].

In our talk we shall explicitly construct a subgroup of F which is isomorphic to $\mathbf{Z} \operatorname{wr} \mathbf{Z}$ and which is first-order definable in F with certain parameters, thereby showing that the elementary theory of Thompson's group F is hereditarily undecidable.

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On Conjugacy Classes of Finite Groups with a regard towards Groups of Squarefree Order

Anna Torstensson

Conjugacy classes are a vital tool in the analysis of groups. A condensed piece of information about a group is its class number, that is its number of conjugacy classes. Starting with small numbers mathematicians have tried to catalogise all groups having a given class number. In my talk I will give brief history of this problem. The starting point was when Landau in 1903 managed to show that there is only a finite number of non-isomorphic finite groups with class number k, and also derived an upper bound on the group size as a function of k. In [6] Poland extends the complete list of groups with class number k from $k \leqslant 5$ to $k \leqslant 7$. Using computerised methods this was later pushed forward to cover $k \leqslant 9$ (references can be found in [3]) and considering only simple groups further to $k \leqslant 12$ by Komissarcik in [3]. The reason to limit the search to simple groups was that for $5 \leqslant k \leqslant 9$ the largest groups with a given class number are all simple. Hence one might guess that this is true also for larger k even though this seems to be unknown.

There has also been successful attempts to attack other types of groups. For example p-groups where it is known that a group of order p^m has $(n + r(p - 1))(p^2 - 1) + p^e$ conjugacy classes where r is some non-negative integer and $m = 2n + e$, $e = 0$ or 1 depending on the parity of m. This result can be found in [7].

For the rest of the talk we will go in the opposite direction and instead of p-groups consider groups of squarefree order. We will look at an explicit formula for the class numbers groups of a given squarefree order can have. As a by product of the analysis we will also see that a group of squarefree order having k conjugacy classes can have no more than k^3 elements. There are bounds on the group order in the general case, but in contrast to this bound they are far from sharp. All these results are proved in detail in [1].

It is of course a substantial confinement to consider only groups of squarefree orders. It should be noted however that most group orders are covered, as

$\lim_{n\to\infty} \frac{Q(n)}{n} = \frac{6}{\pi^2}$, where $Q(n)$ denotes the number of squarefree numbers $\leqslant n$. (See [2].) In addition the techniques developed here can hopefully be modified in order to cover other types of groups which have relatively simple presentations.

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Permutation invariance of binary codes and linearity of Reed-Muller codes over some rings

Bahattin Yıldız

By starting with an open conjecture from [2] in which the authors conjectured that the Reed-Muller codes $RM(r,m)$ won't be $\mathbb{Z}_4$-linear for non-trivial cases, we try to answer the same question for the ring $\mathbb{F}_2 + u\mathbb{F}_2$. Linear codes over this ring have been studied in works like [3], [4] and [5]. In this work we will be proving that the Reed-Muller codes $RM(r,m)$ are $\mathbb{F}_2 + u\mathbb{F}_2$-linear contrary to the case of $\mathbb{Z}_4$. In fact we will generalize this further to some other rings.

The tools that we will be using are going to be relating $\mathbb{F}_2 + u\mathbb{F}_2$-linearity as well as linearity over these other rings that we will consider of binary codes with permutation invariance where we consider some suitable permutation group in each case. These will help us understand these linear codes better.

A linear code over a ring R of length n is just an R-submodule of R^n and a permutation acts on a linear code by switching the coordinates. In this aspect linear codes are algebraic objects and while proving our results we will just use these aspects of them.

The main rings to be used are going to be $\mathbb{F}_2 + u\mathbb{F}_2$ which is constructed via $u^2 = 0$, $\mathbb{F}_2 + u\mathbb{F}_2 + u^2\mathbb{F}_2 + u^3\mathbb{F}_2$ which is constructed via $u^4 = 0$ and its generalizations. We will also be considering the ring $\mathbb{F}_2 + u\mathbb{F}_2 + v\mathbb{F}_2 + uv\mathbb{F}_2$ which is constructed via $u^2 = v^2 = 0, uv = vu$. We will demonstrate that being linear over the ring $\mathbb{F}_2 + u\mathbb{F}_2 + u^2\mathbb{F}_2 + \cdots + u^{2^k-1}\mathbb{F}_2$of a binary code of length $2^k n$ is equivalent to being invariant under the permutation group $\mathbb{Z}_{2^k}$ and we will also show that linearity over the ring $\mathbb{F}_2+u\mathbb{F}_2+v\mathbb{F}_2+uv\mathbb{F}_2$ of a binary code of length $4n$ means being invariant under the permutation group K_4, where K_4 denotes the Klein-4 group. After these results we will finally consider the case of Reed-Muller codes $RM(r,m)$ and we will answer the question posed by the authors in [2] for the rings that we have considered above.

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4 Posters

On the strong Hadamard exponential and logarithm functions

Haci Civciv

This is joint work with Ramazan Türkmen. The strong Hadamard product is considered a powerful matrix multiplication tool for Hadamard and other orthogonal matrices from combinatorial theory. Also, it is well known that the matrix versions of the familiar real and complex exponential and logarithm functions are fundamental for the study of many aspects of matrix group theory. In this note, we first introduce the strong Hadamard exponential and logarithm functions of some special matrices, and then investigate some properties of these functions for these matrices.

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Rational endomorphisms of a nilpotent group

Bounabi Daoud

Let G be a group. A function $f : G \to G$ is said to be *rational* if there exist elements $a_1, \dots, a_r, a_{r+1} \in G$ and integers $h_1, \dots, h_r \in \mathbb{Z}$ such that

$$f(x) = a_1 x^{h_1} a_2 x^{h_2} \cdots a_r x^{h_r} a_{r+1} \text{ for all } x \in G.$$

If the exponents $h_1, \dots, h_r$ can be chosen to be positive, we say that f is *polynomial*; it is the case when for example the group G is of finite exponent. The set $F(G)$ of all rational functions of G forms a monoid with the usual composition of functions, this monoid has two important submonoids, $\mathcal{F}(G)$ the set of rational functions f such that $f(1) = 1$, and $End_r(G)$ the set of rational functions which are endomorphisms. It is obvious that $End_r(G) \subseteq \mathcal{F}(G)$. We denote by $F^*(G), \mathcal{F}^*(G)$, and $End_r^*(G)$, the group of invertible elements of the corresponding monoids. Note that a bijective rational function of a group G is not necessarily in $F^*(G)$ (e.g., consider $G = (0, \infty)$ with multiplication and $f : x \mapsto x^3$). On the other hand, if G is finite, then $F^*(G) = Sym(G) \cap F(G)$, $End_r^*(G) = Sym(G) \cap End_r(G)$, where $Sym(G)$ is the set of all bijections on G.

Following [2], we denote by $J(G)$ the set of integers m such that there exist t integers $m_1, \dots, m_t$ $(t \geqslant 1)$ and t elements $c_1, \dots, c_t$ of G such that $m = m_1 + \cdots + m_t$ and $c_1 x^{m_1} \cdots c_t x^{m_t} = 1$ for all $x \in G$. The set $J(G)$ is an ideal of the ring of integers $\mathbb{Z}$ (see [2, p. 431]). Let $m \geqslant 0$ be the integer such that $J(G) = (m)$ and $f \in F(G)$, where $f(x) = a_1 x^{h_1} \cdots a_r x^{h_r} a_{r+1}$. Put $h = h_1 + \cdots + h_r$ and $h_0 \in \{0, \dots, m-1\}$ is the unique integer such that $h_0 - h \in J(G)$. If $h_0 = h + qm$ for some integer q, then $f(x) = f(x)(c_1 x^{m_1} \cdots c_t x^{m_t})^q$. Thus we may assume that $h \in \{0, \dots, m-1\}$ and we call $\bar{h}$ the *degree* of f. If $f(1) = 1$ (i.e., $f \in \mathcal{F}(G)$), then

$$f(x) = x^h \prod_{i=1}^{r} [x^{h_i}, b_i][x^{h_i}, b_i, x^{h-(h_1+\cdots+h_i)}],$$

where $b_i = (a_1 \cdots a_i)^{-1}$ for all $i \in \{1, \ldots, r\}$.

Endimioni [2] has proved that if G is nilpotent, $F^*(G)$ is a nilpotent-by-(finite abelian) group. In this paper we prove that

Theorem 1 *If G is a group, we denote by $\mathcal{F}_1(G)$ the set of elements o $\mathcal{F}(G)$ of degree $\bar{1}$. Then, if G is nilpotent of class $c \geqslant 2$, we have*

(i) $\mathcal{F}_1(G)$ is a normal subgroup of $\mathcal{F}^*(G)$, and $\frac{\mathcal{F}^*(G)}{\mathcal{F}_1(G)}$ is isomorphic to the group of invertible elements of the ring $\frac{\mathbb{Z}}{J(G)}$.

(ii) $\mathcal{F}_1(G)$ is nilpotent of class $c-1$.

Schweigert [3, Satz 3.5, p.37] showed that if G is a finite group, then G is nilpotent if and only if $End^*_r(G)$ is nilpotent. Corsi Tani and Rinaldi Bonafede [1, Theorem 3.5, p. 288] improved Schweigert's result by proving that if G is a finite group, then it is nilpotent of class $c \geqslant 2$ if and only if $End^*_r(G)$ is nilpotent of class $c-1$. Note that if G is abelian, then $End^*_r(G) = \mathcal{F}^*(G)$ coincides with the group of universal power automorphisms which is contained in the centre of $Aut(G)$, the automorphisms group of G. We generalize [1, Theorem 3.5, p. 288] for infinite groups, in fact we prove

Theorem 2 *Let G be a group and $c \geqslant 3$ be an integer. Then G is nilpotent of class c if and only if $End^*_r(G)$ is nilpotent of class $c-1$. Moreover, $End^*_r(G)$ is normal in $Aut(G)$.*

It was proved in [1, Theorem 5.2, p. 291] that if G is a finite group and $r \geqslant 2$ is a given integer, then the sum of any r inner automorphisms of G is an automorphism of G (i.e., the map $x \mapsto x^{a_1} \cdots x^{a_r}$ is an automorphisms of G for any r elements $a_1, \ldots, a_r \in G$) if and only if G is 2-Engel with the property that $\binom{r}{2} \equiv 0 \bmod G'$ and $\gcd(r, \exp(G)) = 1$. Here we generalize this result to any group.

Let h be an integer. We say that a group G satisfies $\mathcal{I}(h)$ ($\mathcal{E}(h)$) if the map $x \longmapsto (x^{a_1})^{h_1} \ldots (x^{a_r})^{h_r}$ is an invertible rational endomorphism (epimorphism) of G for all $a_1, a_2, \ldots, a_r \in G$ and $h_1, h_2, \ldots, h_r \in \mathbb{Z}$ such that $h = h_1 + \cdots + h_r$.

Theorem 3 *Let G be a group and h an integer different of $-1, 0, 1$. Then G satisfies $\mathcal{I}(h)$ if and only if G is 2-Engel with conditions:*

1) exp(G) is finite and $\gcd(h, exp(G)) = 1$.

2) $exp(G')$ divides $h(h-1)/2$.

If G satisfies $\mathcal{E}(h)$, then G is a 2-Engel group with the only property that $exp(G')$ is finite and divides $h(h-1)/2$.

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On Cofinitely Weak Supplemented Modules

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An R-module M is called weakly supplemented, if every submodule N of M has a *weak supplement*, i.e. a submodule K satisfying $N + K = M$ and $N \cap K \ll M$. Semisimple modules and artinian modules are weakly supplemented. A submodule N of a module M is said to be *cofinite* if M/N is finitely generated. M is called a *cofinitely weak supplemented* (briefly *cws*) module if every cofinite submodule of M has a weak supplement. An R-module M is called *locally noetherian* if every finitely generated submodule of M is noetherian.

Theorem 1 *Let M be a locally noetherian module with $\operatorname{Rad} M \ll M$. The following statements are equivalent.*

(i) M is weakly supplemented.

(ii) Every cyclic submodule of M has (is) a weak supplement.

(iii) Every finitely generated submodule of $M/\operatorname{Rad} M$ is a direct summand.

Theorem 2 *For a locally noetherian module M, the following are equivalent.*

(i) M is a cws*-module.*

(ii) M/X is a cws*-module for some $X \subseteq \operatorname{Rad} M$.*

(iii) $M/\operatorname{Rad} M$ is a cws*-module.*

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On the Singular Elliptic Curves over Finite Fields

Betul Gezer

This is joint work with Osman Bizim and Ahmet Tekcan. We consider the rational points on singular elliptic curves over finite fields $\mathbb{F}_p$. We give results concerning the number of the points on the singular elliptic curves, $y^2 = x^3$ and $y^2 = x^3 + ax^2$ over $\mathbb{F}_p$ where p is a prime. We give the number of solutions to $y^2 = x^3$ and $y^2 = x^3 + ax^2$ over $\mathbb{F}_p$ by means of quadratic residue character, and also for the curve $y^2 = x^3$ we use the cubic residue character. Also some results are given on the sum of x-and y-coordinates of the points (x, y) on these curves. We determine the structure of the group of the rational points and torsion points on these curves and finally we generalize the results concerning the number of points in $\mathbb{F}_p$ to $\mathbb{F}_{p^n}$.

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Characters and Quasi-Permutation Representations of 2-Groups with $|\Omega_2(G)| = 16$

Ghodrat Ghaffarzadeh

By a quasi-permutation matrix we mean a square matrix over the complex field $\mathbb{C}$ with non-negative integral trace. Thus every permutation matrix over $\mathbb{C}$ is a quasi-permutation matrix. For a given finite group G, let $p(G)$ denote the minimal degree of a faithful permutation representation of G (or of a faithful representation of G by permutation matrices), let $q(G)$ denote the minimal degree of a faithful representation of G by quasi-permutation matrices over the rational field $\mathbb{Q}$, and let $c(G)$ denote the minimal degree of a faithful representation of G by complex quasi-permutation matrices. It is easy to see that

$$c(G) \leqslant q(G) \leqslant p(G)$$

where G is a finite group. In [3], finite $2-$groups G have been determined with the property $\mid \Omega_2(G) \mid = 16$. We recall that $\Omega_2(G) = < x \in G : x^4 = 1 >$. In this paper, we study characters and quasi-permutation representations of this groups which are cyclic center and have the additional property that G is non-abelian and has a normal elementary abelian subgroup E of order 8. In [3], these groups have been determined in terms of generators and relations. Also, it is shown that G/E is either cyclic or a generalized quaternion Q_{2^n} of order 2^n, $n \geqslant 3$. If G/E is cyclic, then there exists one class of $2-$groups [3, theorem 3.1], and if G/E is generalized quaternion group, then there exist two classes of $2-$groups [3, theorems 3.3 and 3.4].

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Bernoulli–Frey Elliptic Curves

İlker İnam

This is joint work with Aysun Yurttaş, Osman Bizim, and İ. Naci Cangül.

In this work, authors establish some connections between Bernoulli numbers and Elliptic curves. They form a new class of elliptic curves of Frey type by using Bernoulli numbers as coefficients and call this class Bernoulli-Frey elliptic curves. In the first part, they give a summary about Bernoulli numbers, then in the second part, they introduce elliptic curve theory which became more famous with the solution of Fermat's Last Theorem. Finally, they give a definition of a class of elliptic curves called "Bernoulli-Frey Elliptic Curves". Also, they give a classification of rational points on Bernoulli-Frey elliptic curves and they give some properties of them.

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Application of polytope method to irreducibility of polynomials over $\mathbb{Z}_{p^k}$

Fatih Koyuncu

Ostrowski observed that there is a relation between the factorization of a polynomial $f \in F[x_1, \dots, x_n]$ for any field F and the integral decomposition, with respect to Minkowski sum, of the Newton polytope of f. Here, extending this result to some families of polynomials in $R[x_1, \ldots, x_n]$ for an arbitrary ring R, we have found infinitely many classes of multivariate polynomials which are irreducible over $\mathbb{Z}_n$ for any positive integer n, in particular, over $\mathbb{Z}_{p_i^k}$ for any prime number p_i and arbitrary positive integer k.

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Centralizer of Engel elements in a group

Carmela Sica

This is joint work with G. Endimioni (Université de Provence, Marseille (Fr)).

Let G be a group and H a subgroup. Given some information about the centralizer $C_G(H)$, what can we deduce about G? This type of problem is well-known and has been studied by a number of people, see for instance [1, 3, 5]. We are specially interested in the following result of Onishchuk and Zaĭtsev ([4]).

Proposition 1 *Let $\mathfrak{X}$ denote the class of polycyclic groups, or the class of Černikov groups, or the class of minimax groups, or the class of groups of finite rank (in the sense of Prüfer). Let G be a nilpotent group, H a finitely generated subgroup. Then $C_G(H)$ belongs to $\mathfrak{X}$ if and only if G belongs to $\mathfrak{X}$.*

From now on, the rank of a group is defined in the sense of Prüfer. Thus a group G has finite rank if there exists an integer r such that every finitely generated subgroup of G can be generated with at most r elements; in this case, the least integer r satisfying this condition is the rank of G.

An example given by the authors shows that their result can be false if we consider a *locally* nilpotent group G. Let $A = \langle a_1 \rangle \times \langle a_2 \rangle \times \ldots \times \langle a_n \rangle \times \ldots$ be an infinite elementary abelian p-group and $B = \langle b \rangle$ be an infinite cyclic group. Consider the semidirect product $G = A \rtimes B$, with $a_1^b = a_1$ and $a_i^b = a_{n-1}a_n$ for $n > 1$. Then G is a metabelian locally nilpotent group, with an element, b, whose centralizer is polycyclic (thus minimax and of finite rank), and which is of infinite rank (so it is neither polycyclic, nor minimax). Notice that this example does not show that Proposition 1 fails for the class of Čhernikov groups.

In the paper [2], we aimed at obtaining similar results, for instance when G is soluble. In particular, we strengthen the theorem of Onishchuk and Zaĭtsev when $\mathfrak{X}$ is the class of polycyclic groups or the class of Černikov groups. Obviously, if we just assume that G is soluble, we must formulate some extra hypothesis about H. We can expect reasonably that $C_G(H)$ exercises a strong influence on G when the elements of H are not very far (in some sense) from the central elements. That leads to the notion of Engel element, that can be considered as a generalization of the notion of central element. We recall that in a group G, an element y is said to be a *left Engel* element if for all x in G, there exists an integer $m \geqslant 0$ such that $[x,_m y] = 1$ (as usual, $[x,_m y]$ is inductively defined by $[x,_0 y] = x$ and $[x,_m y] = [x,_{m-1} y]^{-1} y^{-1} [x,_{m-1} y] y$ when $m > 0$). If m can be chosen independently of x, one says that y is a *bounded* left Engel element. We shall write $L(G)$ for the set of left Engel elements of G, and $\overline{L}(G)$ for the set of bounded left Engel elements.

Our first result generalizes Proposition 1 when $\mathfrak{X}$ is the class of polycyclic groups.

Theorem 1 *Let G be a soluble group. Let H be a finitely generated subgroup contained in $\overline{L}(G)$ and suppose that $C_G(H)$ is polycyclic. Then G is polycyclic.*

The example in [4] shows also that Theorem 2 fails if we replace $\overline{L}(G)$ by $L(G)$ in its statement. As we noticed above, Onishchuk and Zaĭtsev did not say if Proposition 1 is still true for locally nilpotent groups in which the centralizer is a Černikov group. Our next theorem gives a positive answer. Before stating this result, recall also that a *radical group* is a group having an ascending series with locally nilpotent quotients. The class of radical groups is very large, containing for instance all soluble groups and all locally nilpotent groups.

Theorem 2 *Let G be a radical group. Let H be a finitely generated subgroup contained in $L(G)$ and suppose that $C_G(H)$ is a Černikov group. Then G is a soluble Černikov group.*

Finally, we have proved that Proposition 1 holds for a *periodic* locally nilpotent group G when $\mathfrak{X}$ is the class of groups of finite rank (with this choice of $\mathfrak{X}$, we have seen that Proposition 1 fails for an arbitrary locally nilpotent group).

Theorem 3 *Let G be a periodic locally nilpotent group. Let H be a finitely generated subgroup such that $C_G(H)$ has finite rank. Then G has finite rank.*

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The Connection Between Principal Indefinite Quadratic Forms and Elliptic Curves

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This is joint work with Betül Gezer and Osman Bizim. Let p be a prime number such that $p \equiv 1(\bmod 4)$, let $\mathbb{F}_p$ be a finite filed. Let $F(x, y) = x^2 + xy + \frac{1-p}{4}y^2$ be the principal indefinite binary quadratic form of discriminant $\Delta = p$ and let $E : y^2 = x^3+x^2+\frac{1-p}{4}x$ be the corresponding elliptic curve over $\mathbb{F}_p$. In this paper we prove that the order of E over $\mathbb{F}_p$ is p if $p \equiv 1(\bmod 8)$ or $p+2$ if $p \equiv 5(\bmod 8)$. Further we obtain some formulas on the sum of $x-$and $y-$coordinates of the points (x, y) on E. Later, we consider the reduction of F. We prove that the reduction of F is $R^2(F) = (1, 1 + 2j, j^2 + j - 2k)$ if $p \equiv 1(\bmod 8)$ or $R^2(F) = (1, 1 + 2j, j^2 + j - 1 - 2k)$ if $p \equiv 5(\bmod 8)$, where $k = \frac{p-1}{8}$ or $\frac{p-5}{8}$, respectively and j is any positive integer such that $k \in A_j$.

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Weakly Supplemented Lattices

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L will mean a complete modular lattice with greatest element 1. An element a in a lattice with zero is called an *atom* if $0 \prec a$, i.e. there is no element $b \in L$ such that $0 < b < a$. A lattice L is called *semiatomic* if 1 is a join of atoms in L. An element c of a complete lattice L is said to be *compact* if for every subset X of L with $c \leqslant \bigvee X$ there exists a finite subset F of X such that $c \leqslant \bigvee F$. A lattice L is called a *compact lattice* if 1 is compact and a *compactly generated lattice* if each of its elements is a join of compact elements. We call an element a of L a *supplement* of an element b if $a \vee b = 1$ and a is minimal with respect to this property. Minimality of a is equivalent to $a \wedge b \ll a/0$. Reducing this equivalent condition to $a \wedge b \ll L$, we get the definition of *weak supplement.* L is said to be *supplemented (*respectively, *weakly supplemented)* if every element a of L has a supplement (respectively, weak supplement) in L. In a lattice L the intersection of all maximal ($\neq 1$) elements is called *the radical* of L, denoted by $r(L)$.

Theorem 1 *If a compactly generated lattice L is weakly supplemented then $1/r(L)$ is semiatomic.*

Corollary 1 *A compactly generated lattice L with $r(L) = 0$ is weakly supplemented if and only if it is semiatomic.*

Corollary 2 *Let L be a compactly generated lattice with small radical. Then L is weakly supplemented if and only if $1/r(L)$ is semiatomic.*

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Locally Graded Quotients of Locally Graded Groups

Maria Tota

The presentation will be about a joint work with Akbar Rhemtulla (University of Alberta, Canada), on locally graded groups.

A group G is said to be locally graded if every nontrivial, finitely generated subgroup of G has a nontrivial finite image. The class $\mathcal{L}$ of locally graded groups is quite wide (locally soluble groups and locally residually finite groups belong to $\mathcal{L}$) and it frequently appears in literature ([1], [2], [3], [4], and the papers cited in these) mainly in the study of groups that do not have infinite, finitely generated simple subgroups.

It can be easily seen that a subgroup of an $\mathcal{L}$-group is an $\mathcal{L}$-group but the class $\mathcal{L}$ is not closed under forming quotients, for free groups are locally graded thus every group can occur as a quotient of an $\mathcal{L}$-group.

Hence, naturally arises the problem of looking for $\mathcal{L}^{QS}$, the $\{Q\text{-}S\}$-closed interior of $\mathcal{L}$, that is the largest subclass of $\mathcal{L}$ which is closed with respect to forming sections.

If we denote by $\mathcal{L}_1$ the class of all groups in which every finitely generated simple section is finite, it turns out that $\mathcal{L}_1 = \mathcal{L}^{QS}$.

Moreover, we give conditions under which a quotient of a locally graded group is locally graded.

In fact, we generalize the result proved in [3], stating that if G belongs to $\mathcal{L}$ and N is a locally nilpotent, normal subgroup of G, then the factor group G/N still belongs to $\mathcal{L}$.

Then, we focus on linear groups which are in particular locally graded and we get more precise conditions for a quotient of a linear group to be locally graded.

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On minimal non torsion-by-nilpotent and non (locally finite)-by-nilpotent groups

Nadir Trabelsi

Denote by $\mathcal{N}$ (respectively, $\mathcal{N}_c$, $\mathcal{F}$) the class of nilpotent (respectively, nilpotent of class at most c, finite) groups and let $\mathcal{X}$ stands for the class of torsion groups or the class of locally finite groups. If Ω is a class of groups, then a group is said to be minimal non-Ω if it is not an Ω-group, while all its proper subgroups belong to Ω. Many results have been obtained on minimal non-Ω, for various classes of groups Ω. In particular, in [1] it is proved that if G is an infinite finitely generated minimal non-$\mathcal{N}$ group, then $G/Frat(G)$ is an infinite simple group, where $Frat(G)$ stands for the Frattini subgroup of G. Also in [3] it is proved that if G is a finitely generated minimal non-$\mathcal{FN}$ group, then G is a perfect group which has no proper subgroup of finite index and $G/Frat(G)$ is an infinite simple group. In this note we obtain a similar result on minimal non-$\mathcal{XN}$ groups and we prove that there are no minimal non-$\mathcal{XN}$ groups which are not finitely generated. More precisely, we prove the following result:

Theorem 1 *If G is a minimal non-$\mathcal{XN}$ group, then G is a finitely generated perfect group which has no proper subgroup of finite index and $G/Frat(G)$ is an infinite simple group.*

Using a well known result of Zaïcev [4] which asserts that an infinite nilpotent group whose proper subgroups are in the class $\mathcal{N}_c$ is itself in $\mathcal{N}_c$, we can see that Theorem 1 has the following consequence:

Corollary 1 *If G is a minimal non-$\mathcal{XN}_c$ group, then G is a finitely generated perfect group which has no proper subgroup of finite index and $G/Frat(G)$ is an infinite simple group.*

Among minimal non-$\mathcal{XN}$ groups and minimal non-$\mathcal{XN}_c$ groups we can cite the example constructed by Ol'shanskii [2] of an infinite simple torsion-free finitely generated group whose proper subgroups are cyclic.

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Radical supplemented modules

E. Türkmen

This is joint work with A. Pancar, and F.Yüzbaşı. Let R be a ring , M a left R - module and U a submodule of M. It is known that V a submodule of M is called a supplement of U in M if U *cap* V << V with U + V = M. We define that V a submodule of M is called a Radical supplement of U in M if U *cap* V $\subseteq$ Rad V with U + V = M. In this paper equality of the radical supplement is shown and various properties of both the radical supplement and the submodule of M has a radical supplement in M are given.Similary, we define that M is called radical supplemented if every submodule U of M has a radical supplement in M. Several properties of the radical supplemented modules are studied.

Theorem 1 *Let M be an R - module and $U, V \leqslant M$. V is a Rad - supplement of U if and only if $U + V = M$ and $< m >= Rm << V$, for each $m \in U \cap V$.*

Theorem 2 *Let M be an R- modul and $U \leqslant M$. If $U << M$, then M is a Rad - supplement of U.*

Lemma 1 *Let M be an R- module and V be a Rad - supplement of U in M. If $K << M$, then $K \cap V \subset RadV$.*

Theorem 3 *Let M be an R- module and V be a Rad - supplement of U in M. If $RadM << M, RadV = V \cap RadM$.*

Theorem 4 *Let M be an R- module and V be a Rad - supplement of U in M. If $L \subset U$, then $(V + L)/L$ is a Rad - supplement of U/L in M/L.*

Theorem 5 *Let M be an R - module and M_1, $M_2 \leqslant M$. If M_1 is Rad supplemented and M_2 is hollow , then $M_1 + M_2$ has a Rad - supplement in M*

Theorem 6 *Let M be an R- module and $K < L < M$.If L is local and L/K is a Rad - supplement in M/K, then L is a Rad - supplement in M.*

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Some special matrix groups under the strong Hadamard product

Ramazan Türkmen

This is joint work with Haci Civciv. Matrix groups are central in many parts of mathematics and its applications, and the theory of matrix groups is ideal as an introduction to mathematics. On the one hand it is easy to calculate and understand examples, and on the other hand the examples lead to an understanding of the general theoretical framework that incorporates the matrix groups. However, the strong Hadamard product has a ring and very pleasing algebra that supports a wide range of fast, elegant and practical algorithms. Several trends in scientific computing suggest that this important matrix operation will have an increasingly greater role to play in the future. First, these product include signal processing, image processing, and quantum computing. Second, the strong Hadamard product is proving to be a very effective way to look at fast linear transforms. In this note, we examine some properties some specific matrix groups under the strong Hadamard multiplication.

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	Tue 22	Wed 23	Thu 24	Fri 25	Sat 26	Sun 27
9.00	Kegel	Stöhr	Hall	Thomas	Passman	Meierfrankenfeld
10.00	P. Smith	Olshanskii	Borovik	Grigorchuk	Lashki	Pragacz
11.00	*coffee*					
11.30	Leinen	Brown	Rowley	Stichtenoth	Shumyatsky	H. Smith
12.30	*lunch*					
14.30	Kuzucuoğlu	Lewis		Shalev	Zelmanov	
15.30	Bryant	Zalesski		Roquette	Pride	
16.00		Papistas			Makarenko	
16.30	*coffee*			*coffee*		
17.00	Amberg	‖ talks		‖ talks	‖ talks	
17.30	Behravesh					
18.00	*reception*					
18.30				*		

Moderators:

	Tue 22	Wed 23	Thu 24	Fri 25	Sat 26	Sun 27
9.00	Ali Nesin	P. Smith	Bilhan	Shumyatsky	Hall	Bryant
14.30	Rowley	Sertöz		Stichtenoth	Zalesski	
17.00 A	Asar	Stöhr		H. Smith	Güloğlu	
17.00 B		Ünlü		Topuzoğlu	Alizade	

* Friday, after dinner: memories of Brian Hartley, with his daughter, C. Barber-Brown

Parallel talks

		17.00	17.30	18.00	18.30
Wed	A	Yalçın	Yalçınkaya	†	
	B	Alizade	Pusat-Yılmaz	Crispin Q.	Kaptanoğlu
Fri	A	Asar	Suleymanova	Diasamidze	
	B	Çeşmelioğlu	Khan	Yıldız	
Sat	A	Levchuk	Ercan	‡	
	B	Basarab	Güçlükan	Altunbulak	Dzhumadil'daev

† Johnson, Arıkan, Torstensson, Russo

‡ Tolstykh, Deloro, Pastori, _______