Antalya Cebir Günleri VII

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18–22 Mayıs 2005

Perge, home of Apollonius

Bu toplantı,
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Antalya Algebra Days VII

May 18–22, 2005

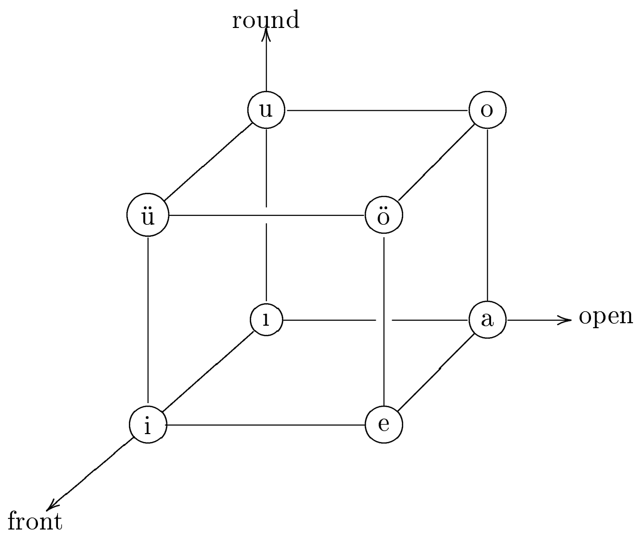
Contents

1 Some notes on the Turkish language

Visitors from abroad may like to know something about the local language; native speakers may wish to check the perceptions of somebody (David Pierce) with mainly an academic knowledge of Turkish:

Developed in 1928 to allow phonetic transcription of the language, the Turkish alphabet has 29 letters. To obtain it from the English alphabet: throw out (Q, q), (W, w), and (X, x); replace the letter (I, i) with the two letters (I, ı) and (İ, i); and introduce the new letters (Ç, ç), (Ğ, ğ), (Ö, ö), (Ş, ş), (Ü, ü).

According to their pronunciation, the eight vowels correspond to the vertices of a cube. At the origin of Cartesian 3-space, place the vowel ı. As fits its simple written form, you pronounce ı by relaxing the mouth completely, but keeping the teeth nearly clenched. The national drink rakı is *not* pronounced like "Rocky": in the last syllable of *this*, the tongue is too far forward. The letter ı is the **close, unround, back** vowel. Other vowels deviate from this by being **open, round** or **front**; let these deviations correspond respectively to movement in the x-, y- or z-directions (right, up, or forwards). Then the vowels can be diagrammed:



In particular: a is like "uh" in English; ö and ü are as in German, or are like the French "eu" and "u"; and u is like the short English "oo". Diphthongs are obtained by addition of y: so, ay is English long "i", and ey is English long "a". Syllables are fairly uniformly stressed.

The consonants that need mention are: c, like English "j"; ç, like English "ch"; ğ, or "soft g", which never begins a word, but lengthens the vowel that precedes it; j, as in French; and ş, like English "sh". Some expressions follow:

Lütfen/Teşekkürler/Bir şey değil (Please/Thanks/It's nothing);
Evet/Hayır (Yes/No); Affedersiniz (You make pardon, *i.e.* Excuse me);
Merhaba (Hello); Günaydın (Day [is] bright, *i.e.* Good morning);
İyi günler/akşamlar/geceler (Good day/evening/night).

Bay/Bayan: Sir/Madam, or gentlemen's/ladies' (room, clothing, &c.);
İtiniz/Çekiniz (Push/Pull); giriş/çıkış (entrance/exit);
sol/sağ (left/right); soğuk/sıcak (cold/hot).

Nasılsınız?/İyiyim; siz?/Ben de iyiyim (How are you?/I'm fine; you?/I'm also fine).

Elinize sağlık: Health to your hand. (This is a standard compliment to the chef, who will reply: Afiyet olsun—May it be healthy.)

Sıfır, bir, iki, üç, dört, beş, altı, yedi, sekiz, dokuz (0,1,2,3,4,5,6,7,8,9);
on, yirmi, otuz, kırk, elli, altmış, yetmiş, seksen, doksan (10, 20, 30, ..., 90);
yüz, bin, milyon, milyar (10^2, 10^3, $(10^3)^2$, $(10^3)^3$);
yüz kırk dokuz milyon beş yüz doksan yedi bin sekiz yüz yetmiş (149 597 870).

Daha/en (more/most); az (less), en az (least).

Kim, ne, ne zaman, nerede, nereye, nereden, niçin, nasıl, kaç, ne kadar?
(who, what, when, where, whither, whence, why, how, how many, how much?)—these Turkish **interrogatives** also function as rudimentary relatives: Ne zaman gelecekler bilmiyorum (What time come-will-they know-don't-I; *i.e.* I don't know when they will come); but most of the work done in English by relative clauses is done in Turkish by verb-forms (participles): "the book that I gave you" in Turkish becomes size verdiğim kitap—you-wards gave-that-I book, or the book *given* to you by me.

In Turkish, you can describe somebody to me for a long time without my having any idea of the sex of that person: there is no **gender**. Even accomplished Turkish speakers of English confuse "he" and "she": there is a unique third-person singular Turkish pronoun, o(n), meaning indifferently "he/she/it".

Turkish is **agglutinative** or **synthetic**. Written as two, but pronounced as one word is the question Avrupa^{0}lı^{1}la^{2}ş^{3}tır^{4}ama^{5}dık^{6}lar^{7}ımız^{8}dan^9 mı^{10}sınız^{11}? Here I have numbered the eleven suffixes. These translate mostly as separate words in English, in the reverse order: Are^{10} you^{11} one-of^9 those^7 whom^6 we^8 could-not^5 *Europeanize* (*i.e.* make^4 be^{2}come^3 Europe^{0}an^1)?

The interrogative particle mısınız here is **enclitic**: in particular, it shows **vowel harmony** with the preceding word. Each syllable of the suffixes above features either a close vowel ({ı, i, u, ü}, which I'll denote #) or an open unround vowel ({a, e}—I'll call it @). Used in a word, an indeterminate vowel # or @ settles down in the vowel-cube to the available point nearest the preceding vowel. Changing "Europeanize" to "Turkify" in the long word above means writing Türkleştiremediklerimizden misiniz?

Agglutination or synthesis can be seen on signs all over: An in^{0}dir^{1}im^2 is an instance^2 of causing^1 to go-down^0, that is, a reduction, a *sale*; while in^{0}il^{1}ir^2 means "is^2 got^1 down-from^0, is an exit [not an entrance]"—it's written at the rear door of city busses. Some common suffixes are:

-ç# (or -c#), indicating a person involved with something: kebapçı (kebab-seller); kilitçi (locksmith); balıkçı (fishmonger); dedikoducu (rumor-monger); gazeteci (journalist or newsagent);

-l#/-s#z, indicating inclusion/exclusion: sütlü/sütsüz (with/without milk); şekerli/şekersiz (sweetened/sugar-free); etli/etsiz (with-meat/meatless);

-l#k, indicating containment or more abstract involvement: tuzluk (salt cellar); kimlik (identity); günlük (daily or diary); gecelik (nightly or nightgown);

-l@r, the ***indefinite*** **plural** marker: it's not used if a definite number is named: başlar (heads); beş baş: (five head); kişiler (people); on iki kişi (twelve person).

Turkish nouns are **declined** roughly as in Latin: there are genitive, *definite* accusative, dative, ablative and locative cases. From gül (rose):
Gülün dikeni; Gülü koparmayın: Rose's thorn; Don't pick *the* rose.
Güle, gülden, gülde: to/from/on (a/the) rose.

Adjectives as such are not declined; but adjectives can be used as nouns. Nouns can indicate **person** in two senses: the person of the *possessor* of the indicated object, and the person *of* the object:
gülüm, gülün, (Deniz'in) gülü: my rose, thy rose, (Deniz's) her-or-his rose;
gülümüz, gülünüz, gülleri: our/your/their rose.
Gülüm. Gülsün. Güldür: I-am/Thou-art/He-She-It-is a rose.
Gülüz. Gülsünüz. Güldürler: We/you/they are a rose.
Güllerdir: They are the roses. Gülümsün: Thou art my rose.

When two nouns are joined, even though the first doesn't name a possessor of the second, the second tends to be put in the third person: bölüm department; matematik bölümü mathematics department. You can see this feature in business names: banka bank; İş Bankası Work Bank.

Not prepositions, but **postpositions** are used: Gül gibi means "like a rose".

There is no verb corresponding to the English **"have".** "I have a rose" becomes Gülüm var—My rose *exists*. The negation of var is yok. But other verbless sentences are negated with değil: "I am not Rose" becomes Gül değilim.

One Turkish verb can comprise an incredible amount of information. From a simple verb-stem like oku- (read), longer stems can be formed with various suffixes. These suffixes can be:

—**vocal**, *i.e.* indicating voice: okun- (be read); okut- (make read); okuttur- (make make read—as might be said of a principal giving orders to his teachers); sev- (love); seviş- (make love);

—**logical**, indicating affirmation, denial, impossibility and their *possibility*:

oku-	read;	okuyabil-	can read;
okuma-	not read;	okumayabil-	may not read;
okuyama-	cannot read;	okuyamayabil-	may be unable to read;

—**modal** and **temporal**: Here are some complete third-person singular verbs (which can stand as complete sentences): okumaktadır ([s/he] is engaged in reading); okumalı (must read); okusun (let [him/her] read); okusa (if only [s/he] would read); okuyor (is reading); okuyacak (will read); okur (reads, is a reader); okumaz (does not read); okudu (did [definitely] read); okumuş (read [in the past, according to present evidence]).

There are two **verbal nouns**: okumak (to read) and okuma (reading).

Different kinds of endings can be combined into one word: okunabilecekti (was going to be readable); okuyamamamımız (our inability to read).

Some sayings: Balcının var bal tası; oduncunun var baltası (A honey-seller has a honey-pot; a woodsman has an axe). Bakmakla öğrenilse, köpekler kasap olurdu: If learning were done by watching, dogs would be butchers.

2 Schedule of talks (long form)

MAY 18, WEDNESDAY

9:00-9:50	Jacob Murre	On properties of the Chow motives of an algebraic surface
10:00-10:50	Keith Nicholson	Clean endomorphism rings
Coffee Break		
11:30-12:20	Toma Albu	Connections of cogalois theory with Clifford extensions, strongly group graded algebras, and Hopf algebras
Lunch Break		
14:00-14:50	Ze-Li Dou	Periods of automorphic forms
Short Break		
Session 1		
15:00-15:30	İlhan İkeda	Jacquet-Zagier theory and the trace formula
15:30-16:00	İsmail Naci Cangül	Rational points on the elliptic curves $y^2 = x^3 + a^3$ in F_p where p is prime
Coffee Break		
16:30-17:00	Muhammed Uludağ	Calabi-Yau orbifolds
17:00-17:30	Meral Tosun	A special Lie algebra and singularities
Short Break		
17:40-18:10	Vincenzo Micale	The Poincaré series of the module of derivations of some monomial rings
18:10-18:40	Bilal Khan	A graphic generalization of arithmetic
Short Break		
18:50-19:20	Vladimir M. Levchuk	Functions on certain finite simple groups
Session 2		
15:00-15:30	Naim Çağman	Operations on soft sets
15:30-16:00	Gökçen Alptekin	On norms of circulant and semi-circulant matrices with the Pell and Pell-Lucas numbers
Coffee Break		
16:30-17:00	Çiğdem Özcan	Semiperfect modules with respect to a fully invariant submodules
17:00-17:30	Nil Orhan	Cojective modules in the class of $B(M, X)$
Short Break		
17:40-18:10	Ferruh Özbudak	Some recent results on codes and curves
18:10-18:40	Fatih Koyuncu	Absolute irreducibility of polynomials by the Newton polytope method
Short Break		
18:50-19:20	Şükrü Yalçınkaya	Recognition of the p-core in finite groups

MAY 19, THURSDAY

9:00-9:50	Anthony Scholl	TBA
10:00-10:50	James Lewis	Algebraic Cycles and Mumford-Griffiths Invariants
Coffee Break		
11:30-12:20	Vasudevan Srinivas	Zero cycles and complete intersection points on affine varieties
Lunch Break		
14:00-14:50	Robert Hartmann	Young modules and filtration multiplicities for Brauer algebras
Short Break		

Session 1

15:00-15:30	Vladimir Tolstykh	On Bergman's property for the automorphism groups of relatively free groups
15:30-16:00	Ayşe Berkman	Simple L^*-groups of finite Morley rank
Coffee Break		
16:30-17:00	Salih Azgın	
17:00-17:30	David Pierce	Differential fields of any characteristic
Short Break		
17:40-18:10	Tsetska Rashkova	Description of superinvolutions in $M(4)$
18:10-18:40	K. M. Rangaswamy	Mixed modules of finite torsion-free rank over a valuation domain
Short Break		
18:50-19:20	Alexander A. Fomin	A category of matrices representing two categories of abelian groups

Session 2

15:00-15:30	Rafail Alizade	On lambda- and mu- dimensions of modules
15:30-16:00	Engin Mermut	The inductive closure of the proper class of supplements in abelian groups
Coffee Break		
16:30-17:00	Özgün Ünlü	Free group actions on products of spheres
17:00-17:30	Laurence Barker	The semisimple Mackey algebra is a direct product of Hecke algebras
Short Break		
17:40-18:10	Alireza Abdollahi	Non-commuting graph associated with a graph
18:10-18:40	Ali Madanshekaf	Quotient of PQ-hyperstructures
Short Break		
18:50-19:20	Ergün Yalçın	Generalized Burnside rings and group cohomology

Evening Class

20:30-21:30	Yusuf Civan	Torsal varyeteler I

MAY 20, FRIDAY

9:00-9:50	Patrick F. Smith	Artinian ring and modules
10:00-10:50	Robert Wisbauer	Coprime comodules and corings
Coffee Break		
11:30-12:20	William Wickless	Some basic algebraic questions as illustrated by a class of mixed abelian groups
Lunch Break		
14:00-19:00	**Excursion**	
Evening Class		
20:30-21:30	Özgür Kişisel	Torsal varyeteler II

MAY 21, SATURDAY

9:00-9:50	Vesselin Drensky	Computing with matrix invariants
10:00-10:50	V. D. Mazurov	A characterization of alternating groups
Coffee Break		
11:30-12:20	Simon Thomas	Property tau and the classification problem for the torsion-free abelian groups of rank 2
Lunch Break		
14:00-14:50	Matthew Kerr	Higher Abel-Jacobi maps and elliptic functions
Session 1		
15:00-15:30	Gizem Karaali	Dynamical quantum groups, the super story
15:30-16:00	Yıldıray Ozan	Relative flux homomorphism in symplectic geometry
Coffee Break		
16:30-17:00	Hayrullah Ayık	The structure of elements in finite full transformation semigroups
17:00-17:30	Gonca Ayık	On factorizations and generators in transformation semigroups
Short Break		
17:40-18:10	Recep Şahin	Extended Hecke groups and their some normal subgroups
18:10-18:40	Müge Kanuni	A short survey on modules and incidence rings

Session 2 (Graduate Student Session)

15:00-15:20	Ergün Yaraneri	On Mackey algebras: Clifford theory and group gradings
15:20-15:40	Engin Büyükaşık	On weakly supplemented modules
15:40-16:00	Eylem Toksoy	Absolutely supplement modules
Coffee Break		
16:30-16:50	Olcay Çoşkun	Mackey functors, restriction functors, conjugation functors and transfer functors
16:50-17:10	Fatma Altunbulak	Some remarks on a theorem of Jon F. Carlson on filtrations of modules
17:10-17:30	Aslı Güçlükan	T-power of a G-set and the exponential map of Burnside rings
Short Break		
17:40-18:00	Ali Öztürk	Homology of real algebraic varieties and morphism to sphere
18:00-18:20	Caner Koca	Orbits in the anti-invariant sublattice of the K3 lattice
18:20-18:40	Daniela Ferrarello	Ideals and graphs

Evening Class

18:50-19:40	Meral Tosun	Torsal varyeteler III

MAY 22, SUNDAY

9:00-9:50	Richard M. Thomas	Formal languages and groups
10:00-10:50	Piotr Pragacz	Bezoutians, Euclidean algorithm, and orthogonal polynomials
Coffee Break		
11:30-12:20	Alexander Klyachko	Quantum marginal problem and representations of the symmetric group

Lunch and Farewell

3 Abstracts

—ordered alphabetically by the last name of the speaker:

Non-commuting graph associated with a group

Alireza Abdollahi

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Let G be a non-abelian group and let $Z(G)$ be the center of G. Associate a graph Γ_G (called the non-commuting graph of G) with G as follows: Take $G\backslash Z(G)$ as the vertices of Γ_G and join two distinct vertices x and y, whenever $xy \neq yx$. We want to explore how the graph theoretical properties of Γ_G can effect on the group theoretical properties of G. We conjecture that

If G and H are two non-abelian finite group such that $\Gamma_G \cong \Gamma_H$, then $|G| = |H|$.

We prove that if the latter conjecture is true for solvable AC-groups, then it is true for all groups, where a group G is called an AC-group if $C_G(x)$ is abelian for all $x \in G\backslash Z(G)$. Among other results we show that if G is a finite non-abelian nilpotent group and H is a group such that $\Gamma_G \cong \Gamma_H$ and $|G| = |H|$, then H is nilpotent. We give some groups with unique non-commuting graph, i.e. groups G with the property that if $\Gamma_G \cong \Gamma_H$ for some group H then $G \cong H$. As it expects (and we will show it) the non-commuting graph of a group, in general, is not unique and there are non-isomorphic groups with the same non-commuting graph. But it is shown that some of non-abelian finite simple groups have unique non-commuting graph it is proved e.g., for Suzuki simple groups and $\mathrm{PSL}_2(q)$ $(n > 2)$. In view of these results, we state the following conjecture:

Let S be a finite non-abelian simple group and G is a group such that $\Gamma_G \cong \Gamma_S$. Then $G \cong S$.

References

[1] A. Abdollahi, S. Akbari and H.R. Maimani, *Non-commuting graph of a group*, Submitted.

[2] A. Abdollahi, A. Akbari and H. Dorbidi, *Non-commuting graph of a finite non-abelian simple group*, In preparation.

Connections of Cogalois Theory with Clifford extensions, graded algebras, and Hopf algebras

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The aim of this talk is to present to a general audience some interesting connections of *Cogalois Theory* with *Clifford extensions, strongly group graded algebras*, and *Hopf algebras*.

Cogalois Theory is a fairly new theory that investigates field extensions, finite or not, possessing a Cogalois correspondence. This theory is somewhat dual to the very classical one known as *Galois Theory* investigating field extensions possessing a Galois correspondence.

The concepts of *Clifford system* and *Clifford extension* were invented in 1970 by *Everett C. Dade* in two papers appeared in *Annals of Mathematics* devoted to the so called *Clifford Theory*. This theory investigates when an absolutely irreducible character of a normal subgroup N of a finite group G, defined over an algebraically closed field of arbitrary characteristic, can be extended to a character of G. Dade also introduced ten years later in a paper in *Mathematische Zeitschrift* the concept of *strongly group graded algebra*.

In this talk we analyze first the basic concepts of *Cogalois Theory* like *G-radical, G-Kneser*, and *G-Cogalois field extension* in terms of *Clifford extensions* and *strongly group graded algebras*. We describe then the Kneser and Cogalois field extensions in terms of *Galois H-objects* appearing in *Hopf algebras*.

On $\lambda-$ and $\mu-$ Dimensions of Modules

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A module will mean a unitary left R-module over an arbitrary but fixed ring R with identity. Let $(\mathcal{F},\mathcal{C})$ be a *cotorsion theory*, i.e. a pair of modules such that $\mathcal{F}=\left\{F:\operatorname{Ext}^1(F,C)=0\ \forall\ C\in\mathcal{C}\right\}$ and $\mathcal{C}=\left\{C:\operatorname{Ext}^1(F,C)=0\ \forall\ F\in\mathcal{F}\right\}$. A *partial $\mathcal{F}$-resolution* (or *partial $\mathcal{F}$-projective resolution*) of a module M of length n is a complex $F_n\overset{d_n}{\to}F_{n-1}\to\ldots\to F_1\overset{d_1}{\to}F_0\overset{d_0}{\to}M\to 0$ with each $F_i\in\mathcal{F}$, which is $\operatorname{Hom}(F,-)$ exact for every $F\in\mathcal{F}$. Similarly a *partial $\mathcal{C}$-resolution* of a module M of length n is a $Hom(-,\mathcal{C})$ exact complex $0\to M\overset{e_0}{\to}C_0\overset{e_1}{\to}C_1\to\ldots\to C_{n-1}\overset{e_n}{\to}C_n$ with each $C_i\in\mathcal{C}$. If $Ker\ (d_i)\in\mathcal{C}$ for all i, then the partial $\mathcal{F}$-resolution is called *special* and similarly the partial $\mathcal{C}$-resolution above is *special* if $Coker\ (e_i)\in\mathcal{F}$ for all i. If there is a partial $\mathcal{F}$-resolution (special partial $\mathcal{F}$-resolution) of M of length n and there is no longer such complex, we say that the *λ-dimension* (*$\overline{\lambda}$-dimension*) of M is n: $\lambda(M)=n$ ($\overline{\lambda}(M)=n$). If there exists a partial $\mathcal{F}$-resolution (special partial $\mathcal{F}$-resolution) for every $n\geq 0$ we say that $\lambda(M)=\infty$ ($\overline{\lambda}(M)=\infty$). The *$\mu$-dimension* (*$\overline{\mu}$-dimension*) is defined dually by means of partial (special partial) $\mathcal{C}$-resolution (see [1]). We say that the cotorsion theory $(\mathcal{F},\mathcal{C})$ satisfies the *Hereditary Condition (HC)*, if $Ext^2(F,C)=0$ for every $F\in\mathcal{F}$, $C\in\mathcal{C}$ (see [2]). $(\mathcal{F},\mathcal{C})$ satisfies the *Strong Hereditary Condition (SHC)*, if $Ext^n(F,C)=0$ for every $F\in\mathcal{F}$, $C\in\mathcal{C}$ and $n\geq 1$.

Theorem 1. *If the cotorsion theory $(\mathcal{F},\mathcal{C})$ satisfies HC and $0\to M'\to M\to M''\to 0$ is exact then $\overline{\lambda}(M)\geq\min\left(\overline{\lambda}(M'),\overline{\lambda}(M'')\right)$.*

Theorem 2. *If the cotorsion theory $(\mathcal{F},\mathcal{C})$ satisfies HC and $\overline{\lambda}(M)=\infty$ then there is an infinite special $\mathcal{F}$-resolution $\ldots\to F_n\overset{d_n}{\to}F_{n-1}\to\ldots\to F_1\overset{d_1}{\to}F_0\overset{d_0}{\to}M\to 0$ of M.*

Theorem 3. *Suppose that the cotorsion theory $(\mathcal{F},\mathcal{C})$ satisfies* SHC *and $gl.dimR=n<\infty$. If $\lambda(M)\geq n-1$ ($\overline{\lambda}(M)\geq n-1$) , then $\lambda(M)=\infty$ ($\overline{\lambda}(M)=\infty$).*

The dual results hold for for the $\mu-$ and $\overline{\mu}-$ dimensions of modules.

Joint work with: Karen Akıncı.

References

[1] E. Enochs, O. M. G. Jenda, *Relative homological algebra*, (Walter de Gruyter, New York, 2000)

[2] K. D. Akıncı, and R. Alizade, Special precovers in cotorsion theories. *Proc. Edin. Math. Soc.* **45** (2002), 411-420.

On Norms of Circulant and Semicirculant Matrices with the Pell and Pell-Lucas Numbers

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[Joint work with Dursun Taşçı, Gazi University, dtasci@gazi.edu.tr]

Let P_n and Q_n be Pell and Pell-Lucas numbers respectively. Let $c_{ij} = a_{j-i \pmod n}$ and $s_{ij} = \begin{cases} a_{j-i+1} & , i \leq j \\ 0 & , i > j \end{cases}$ be ij^{th} entries of $C(a) = (c_{ij})_{n\times n}$ and $S(a) = (s_{ij})_{n\times n}$. $C(a) = (c_{ij})_{n\times n}$ and $S(a) = (s_{ij})_{n\times n}$ are called Circulant and semicirculant matrices.

In the first section, we introduce circulant and semicirculant matrices and give definitions of matrix norms. Also we introduce Pell, Pell-Lucas numbers and MinMax sequences for Pell numbers. In the second section, we give the properties of Pell and Pell-Lucas numbers. In the third section, we define the circulant and semicirculant matrices with the Pell and Pell-Lucas numbers and investigate the eigenvalues, determinants and norms of these matrices.

References

[1] P. J. Davis, Circulant Matrices, John Wiley & Sons, New York, Chichester, Brisbane, 1979.

[2] A. F. Horadam, Applications of Modified Pell Numbers to Representations, Ulam Quarterly, Vol:3, N.1(1994), 34-53.

[3] D. Kalman, R. Mena, The Fibonacci Numbers-Exposed, Mathematics Magazine, Vol:76, N.3(2003), 167-181.

[4] H. Karner, J. Schneid, C. W. Ueberhuber, Spectral Decomposition of Real Circulant Matrices, Lin. Alg. Appl., 367(2003), 301-311.

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[6] R. Melham, Sums Involving Fibonacci and Pell Numbers, Portugaliae Mathematica, Vol:56, N.3(1999), 309-317.

Some Remarks on a Theorem of Jon F. Carlson on Filtrations of Modules

Fatma Altunbulak

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The talk will be a presentation of a paper joint with Ergün Yalçın. We give an alternative proof to a theorem of Jon F. Carlson [1] which states that if G is a finite group and k is a field of characteristic p, then any kG-module is a direct summand of a module which has a filtration whose sections are induced from elementary abelian p-subgroups of G. We also prove two new theorems which can be considered as generalizations of Carlson's theorem.

References

[1] J.F. Carlson, *Cohomology and induction from elementary abelian subgroups*, Quarterly J.Math 51 no 2.(2000)**169-181**

On Factorisations and Generators in Transformation Semigroups

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Let T_n be the full transformation semigroup, which is a semigroup of all self maps of the finite set $X_n = \{1, 2, \ldots, n\}$, and let $T_{n,r} = S_n \cup K_{n,r}$ where S_n be the symmetric group and $K_{n,r}$ is the set of all maps $\alpha : X_n \to X_n$ such that $|im(\alpha)| \leq r$. The classical decomposition of permutations into disjoint cycles can be extended to more general mappings by means of path-cycles. In this talk we first describe an algorithm of decomposition for any element of the full transformation semigroup T_n. Then, by using this algorithm, we give some information about generating sets for the semigroup of all singular self maps of X_n. In addition, the smallest number of elements of $K_{n,r}$ which, together with S_n, generate $T_{n,r}$ is $p_n(r)$, the number of partition of n with r terms.

The results presented in my talk have been obtain in collaboration with John M. Howie (University of St Andrews) and Hayrullah Ayık (Çukurova University).

The Structure of Elements in Finite Full Transformation Semigroups

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The aim of this talk is to describe a natural factorisation of elements in T_n and to demonstrate some applications of this factorisation.

The *index* and *period* of an element a of a finite semigroup are the smallest values of $m \geq 1$ and $r \geq 1$ such that $a^{m+r} = a^m$. An element with index m and period 1 is called an *m-potent* element. Let $X_n = \{1, 2, \ldots, n\}$, and denote the semigroup of all self maps of X_n by T_n. For each $\alpha \in T_n$ we define $\text{fix}(\alpha)$ as $\{x \in X_n \mid x\alpha = x\}$, and we denote the set $X_n \setminus \text{fix}(\alpha)$ by $\text{Shift}(\alpha)$.

It is shown by Gonca Ayık, Hayrullah Ayık, John M. Howie and Yusuf Ünlü that, for an element α of a finite full transformation semigroup with index m and period r, there exists a unique factorisation $\alpha = \sigma\beta$ such that $\text{Shift}(\sigma) \cap \text{Shift}(\beta) = \emptyset$, where σ is a permutation of order r and β is an m-potent. By using this factorisation, the numbers of some kinds of elements in T_n are obtained.

The semisimple Mackey algebra is a direct product of Hecke algebras

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One of the origins of the notion of a Mackey functor is through the work of Yoshida and others on the use of Hecke algebras (endomorphism algebras of permutation modules) in group cohomology. The cohomological Mackey functors are precisely the modules of a certain Hecke algebra. Generally, the Mackey functors are precisely the modules of a more complicated algebra called the Mackey algebra. In applications to group representation theory, the ring of coefficients is often a field of characteristic zero, and the Mackey algebra is semisimple. We shall realize the Mackey algebra, in this case, as a direct product of Hecke algebras.

Simple L^*-Groups of Finite Morley Rank

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This talk (representing joint work with Alexandre Borovik) is about another step towards proving the Cherlin–Zilber Conjecture, which states that every infinite simple group of finite Morley rank is an algebraic group over an algebraically closed field.

An infinite simple group of finite Morley rank is said to be of *even type* if its Sylow 2-subgroups are infinite and of bounded exponent. Such a group is called an *L^*-group*, if every proper simple definable connected section of the group is a Chevalley group over an algebraically closed field of characteristic 2 or a group of degenerate type.

Theorem. *Let G be a simple L^*-group of finite Morley rank and even type, and S a 2-Sylow° subgroup of G. Assume that there is a p-torus of Prüfer rank at least 3 in G normalizing S. Then G is isomorphic to a Chevalley group over an algebraically closed field of characteristic 2.*

Let $\mathcal{M}$ stand for the collection of 2-local° subgroups of G (that is, of the form $N_G^\circ(U)$ for a non-trivial definable connected 2-subgroup U) containing the connected component of the normalizer of a fixed 2-Sylow° subgroup of G as a proper subgroup and minimal with respect to these properties.

Theorem. *Let G be a simple L^*-group of finite Morley rank and even type, and S a 2-Sylow° subgroup of G. Assume that the size of $\mathcal{M}$ is at least 3, and for every $P_1, P_2 \in \mathcal{M}$, $O_2(\langle P_1, P_2\rangle) \neq 1$. Then G is isomorphic to a Chevalley group over an algebraically closed field of characteristic 2.*

On Weakly Supplemented Modules

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Let R be an associative ring with identity and M be a left R-module. A submodule N of M has a weak supplement in M if $N + K = M$ and $N \cap K << M$ for some submodule K of M. M is weakly supplemented if every submodule of M has a weak supplement in M.

In this paper, we prove that under a certain condition, extension of a weakly supplemented module by a weakly supplemented module is weakly supplemented. As a consequence, we obtain some results over some certain rings.

Theorem 1. *Let $0 \to L \to M \to N \to 0$ be a short exact sequence. If L and N are weakly supplemented and L has a weak supplement in M then M is weakly supplemented.*

Proposition 2. *Let R be a noetherian semilocal ring with Jacobson radical J and M be an R-module. Then the following hold.*

1. *If JM has a weak supplement in M and J^nM is weakly supplemented for some $n \in \mathbf{N}$, then M is weakly supplemented.*

2. *Suppose either $J^nM << M$ or J^nM is weakly supplemented and has a weak supplement in M then M is weakly supplemented.*

Proposition 3. *Let R be a Dedekind domain and M be an R-module. Then the following hold.*

1. *If $T(M)$ has a weak supplement in M then M is weakly supplemented if and only if $T(M)$ and $M/T(M)$ are weakly supplemented.*

2. *If Rad$(T(M)) << M$ then M weakly supplemented if and only if $T(M)$ has a weak supplement in M and $M/T(M)$ is weakly supplemented.*

3. *Suppose either R is semilocal or M is torsion. If Rad(M) has a weak supplement in M then every submodule of M is weakly supplemented if and only if Rad(M) is weakly supplemented.*

Joint work with: Rafail Alizade, Izmir Institute of Technology.

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Rational Points on Elliptic Curves $y^2 = x^3 + a^3$ in F_p where p is prime

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In this talk, we consider the rational points on Bachet elliptic curves $y^2 = x^3+a^3$ over finite fields F_p.

Elliptic curves played an important role in the celebrated proof of Fermat's Last Theorem. We begin our talk with a brief history of the relation between elliptic curves, Taniyama-Shimura Conjecture and Fermat's Last Theorem.

We than give some basic information on the general elliptic curves $y^2 = x^3 + Ax+B$ over finite fields F_p with characteristic greater than 3, and their additive grup structure. As our primary concern in this talk is the rational points on these elliptic curves, we recall the known results such as Mordell, Mazur and Siegel theorems.

We show that there are two different classes of values of the prime p, those congruent to 1 and those to 5 modulo 6, once we consider the rational points on these curves. There are many differences between the results corresponding to these two classes because of the methods used in the proofs which depend on the quadratic and cubic residues. The results we give here include the sum of the abscissae of the rational points, the number of these points on each curve and on the whole family of these curves.

We generalise all these results to the elliptic curves $y^2 = x^3 + a^3$ over finite fields F_{p^n}.

This is joint work with Musa Demirci, Gökhan Soydan & Nazlı Yıldız İkikardeş.

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Mackey functors, restriction functors, conjugation functors and transfer functors

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Mackey functors were introduced by Green to provide a unified treatment of group representation-theoretic constructions involving restriction, conjugation and transfer. A fundamental tool is the classification of simple Mackey functors by Thévenaz and Webb [2]. A major application is the canonical induction due to Boltje [1], who also considered restriction and conjugation functors. Introducing transfer functors so as to exhibit some dualities, we realize some of Boltje's constructions in module theoretic terms, and thereby give two new descriptions of the simple functors, and quick proofs of the Thévenaz–Webb classification theorem and semisimplicity theorem.

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Operations on Soft Sets

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The main purpose of this paper is to study the basic notions of the theory of soft sets initiated by Molodtsov [2] as a new mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. We introduce the operations of the soft sets theory to define soft groups and soft subgroup of a soft group with their basic properties in the fuzzy environment. We then discuss some problems of the future.

This is joint work with Hacı Aktaş (Gaziosmanpaşa University).

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Periods of automorphic forms

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Periods of automorphic forms appear in various guises. They arise, for example, cohomologically as certain invariants via a comparison between two distinct $\overline{\mathbb{Q}}$-rational structures a certain space of automorphic forms is endowed with. There are correspondences among the different spaces of automorphic forms. In their most general form, these correspondences are encompassed within the functoriality principles of the Langlands Program, as explained by Professor K. I. Ikeda in the Antalya Algebra Days 2004. Relations among the period invariants, therefore, are expected, and can be established by close examinations of such correspondences.

On the other hand, the period invariants also appear as special values of L-functions associated with automorphic forms, and as integrals over cycles of certain algebraic varieties, and thus are imbued with arithmetic and geometric information. In a celebrated network of conjectures by G. Shimura, relations among the period invariants are mapped out precisely. Such period relations should provide essential data in the description of the underlying motives, the comprehensive theory for which is also largely conjectural presently.

Therefore the investigation of period relations is closely related to the most central questions in number theory today. In this lecture, we shall begin with a brief introduction to the period invariants, and Shimura's period conjectures will be explained together with relevant known results. More recent progress will take up the second part of the talk. Various results of arithmetic interest will be presented, including period relations. No technical proofs will be included in this talk; instead, motivation for questions and the meaning of the results will be emphasized.

Computing with matrix invariants

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Let K be any field of characteristic 0 and let $X_i = \left(x_{pq}^{(i)}\right)$, $p, q = 1, \ldots, n$, $i = 1, \ldots, d$, be d generic $n \times n$ matrices. The conjugation of X_i with the invertible $n \times n$ matrix g,

$$X_i = \left(x_{pq}^{(i)}\right) \to gX_ig^{-1} = \left(y_{pq}^{(i)}\right),$$

defines an action of the general linear group $GL_n = GL_n(K)$ on the polynomial algebra in n^2d variables

$$\Omega_{nd} = K[x_{pq}^{(i)} \mid p, q = 1, \ldots, n,\ i = 1, \ldots, d]$$

by $g * x_{pq}^{(i)} = y_{pq}^{(i)}$, $g \in GL_n$. The algebra $C_{nd} = \Omega_{nd}^{GL_n}$ of the invariants under the action of GL_n by simultaneous conjugation of d matrices of size $n \times n$ consists of all polynomials $f(x_{pq}^{(i)}) \in \Omega_{nd}$ such that

$$g * f(x_{pq}^{(i)}) = f(g * x_{pq}^{(i)}) = f(x_{pq}^{(i)})$$

for all $g \in GL_n$. It is known that C_{nd} is generated by traces of products of generic matrices $\mathrm{tr}(X_{i_1} \cdots X_{i_k})$ and C_{nd} is called also the pure (or commutative) trace algebra. Another related object is the mixed (or noncommutative) trace algebra T_{dn} generated by $X_1, \ldots, X_d$ and C_{nd} regarding the elements of C_{nd} as scalar matrices. The algebra T_{nd} is also known as the algebra of matrix concominants and consists of the invariant functions under a suitable action of GL_n. The algebras C_{nd} and T_{nd} have many applications not only to invariant theory, but also to theory of algebras with polynomial identities, and theory of finite dimensional division algebras.

Traditionally, a result giving the explicit generators of the algebra of invariants of a linear group G is called a first fundamental theorem of the invariant theory of G and a result describing the relations between the generators is a second fundamental theorem. Classical invariant theory gives that the algebra C_{nd} is finitely generated and T_{nd} is a finitely generated C_{nd}-module. More precise results on invariant theory give that there exists a subalgebra S_C of C_{nd} which is isomorphic to a polynomial algebra and such that C_{nd} is a finitely generated free S_C-module, similarly for T_{nd} and some polynomial subalgebra S_T of C_{nd}.

For a fixed n an upper bound for the degree k of the generators of C_{nd} is given in terms of PI-algebras. By the Nagata-Higman theorem the nil algebras of bounded index are nilpotent, i.e., the polynomial identity $x^n = 0$ implies the identity $x_1 \cdots x_m = 0$. Then $k \leq m$ and for d sufficiently large this bound is sharp. As a C_{nd}-module, T_{nd} is generated by the products $X_{j_1} \cdots X_{j_l}$, $l \leq m-1$. A description of the defining relations of C_{nd} is given in the theory of Razmyslov and Procesi in the language of ideals of the group algebras of symmetric groups.

The algebras C_{nd} and T_{nd} are naturally graded and their Hilbert (or Poincaré) series are defined as the formal power series

$$H(C_{nd}, t_1, \ldots, t_d) = \sum \dim C_{nd}^{(k)} t_1^{k_1} \cdots t_d^{k_d},$$

where $C_{nd}^{(k)}$ is the homogeneous component of C_{nd} of multidegree $k = (k_1, \ldots, k_d)$, similarly for the Hilbert series of T_{nd}. Invariant theory of classical groups gives expressions for the Hilbert series in terms of multiple integrals which can be evaluated in principle, but the explicit formulas are given for $n = 2$ and any d, $n = 3$, $d \leq 3$ and $n = 4$, $d = 2$. These Hilbert series are symmetric functions and decompose as infinite linear combinations of Schur functions $S_\lambda(t_1, \ldots, t_d)$. Since the Hilbert series play the role of characters of GL_d and the Schur functions are the characters of the corresponding irreducible GL_d-modules, the multiplicities $m_\lambda(C_{nd})$ and $m_\lambda(T_{nd})$ can be obtained from the Hilbert series of C_{nd} and T_{nd}. For $d \geq n^2$, the multiplicities of the Hilbert series of C_{nd} and T_{nd} can serve as sufficiently good estimates for the multiplicities of the S_N-cocharacter of the multilinear polynomial identities of degree N of the matrix algebra $M_n(K)$.

In this talk we survey some results, old or recent, obtained by a dozen of mathematicians, on minimal sets of generators and the defining relations of C_{nd} and T_{nd}, and on the multiplicities of the Hilbert series of these algebras. The picture is completely understood only in the case $n = 2$. Besides, explicit minimal sets of generators of C_{nd} are known for $n = 3$ and any d and for $n = 4$, $d = 2$. The multiplicities of the Hilbert series of C_{nd} and T_{nd} are obtained only for $n = 3, 4$ and $d = 2$. For $n > 2$ most of the concrete results are obtained with essential use of computers.

Ideals and Graphs

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There are several binomial and monomial ideals one can associate to a graph. The aim of this work, joint with Giuseppa Carra' Ferro, is to use algebraic tools, and in particular Gröbner bases, to discover properties of a graph and to implement procedures (we did it with Maple) in order to obtain these properties automatically. What is known is a correspondence between even cycles and polynomials in a certain binomial ideal. Here we find correspondences between odd cycles and polynomials in an extended binomial ideal. Such results are used in order to show decision procedures for bipartite graphs, different from the usual approaches. Finally topics on monomial ideals and known results in combinatorics are used in order to show decision procedures for minimal vertex covers and cliques of a graph with commutative algebra tools.

A Category of Matrices Representing two Categories of Abelian Groups

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A new category is constructed. Objects are rectangle matrices over some rings generalizing rings of p-adic integers, morphisms are pairs of square matrices. The category is dual to the category of torsion-free finite-rank abelian groups with quasi-homomorphisms ($\mathcal{TFFR}$) and it is equivalent to the category of quotient divisible mixed abelian groups with quasi-homomorphisms ($\mathcal{QD}$). This result has been obtained together with Professor O.Mutzbauer (the Wuerzburg University) and it may be considered as a remake of the well-known classic description by Kurosh-Malcev-Derry.

The duality between $\mathcal{TFFR}$ and $\mathcal{QD}$ has been proved in [1]. The notion of the quotient divisible mixed group has been introduced in the same paper as a generalization of the classic notion of the torsion-free quotient divisible group by R.Beaumont and R.Pierce [2].

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T-Power of a G-set and the exponential map of Burnside rings

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This is a joint work with E. Yalçın. Let G be a finite group, and X be a G-set. We define $T^i(X)$, the i-th T-power of X, as the set of surjective functions from X to the set $\{1, 2, ..., i+1\}$. We completely describe the generating function for the T-power as a polynomial with coefficients in Burnside ring $B(G)$. The motivation for this work comes from a similar calculation done by P. Webb [1] for symmetric and exterior powers of G-sets. As a consequences of our calculation, we obtain a combinatorial description for the exponential map of Burnside rings, and recover some of the results on the exponential map given earlier in [2].

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Young modules and filtration multiplicities for Brauer algebras

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The Brauer algebra $B = B_k(r, \delta)$, originally called the double centralizer algebra, had been introduced by Richard Brauer to replace the symmetric group algebra in classical Schur-Weyl duality, when the general linear group is replaced by the orthogonal or the symplectic group. It has a basis consisting of diagrams, and multiplication is, up to a scalar, given by concatenation. It contains the symmetric group algebra kS_r both as a subalgebra and a quotient. Its representation theory shows similar features as the one of kS_r, though it is more complicated in some aspects. We show how to define permutation modules and Young modules - these are summands of permutation modules - for this algebra, and we also prove a result on filtration multiplicities, similar to a recent theorem by Hemmer and Nakano for the symmetric group algebra. If time allows, I will also outline how our methods apply to other diagram algebras (for example the rook monoid or the Temperley-Lieb algebra). This is joint work with Rowena Paget.

A Short Survey on Modules and Incidence Rings

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For R a commutative ring with identity and X a partially ordered set, we define $I(X,R)$, the incidence ring, to be the set of functions $f : X \times X \longrightarrow R$ such that $f(x,y) = 0$ unless $x \leq y$, with the following operations

$$(f+g)(x,y) = f(x,y) + g(x,y)$$

$$fg(x,y) = \sum_{x \leq z \leq y} f(x,z)g(z,y)$$

For all $f, g \in I(X,R)$ and $x, y \in X$.

In this survey, we consider some results about prime, multiplicative, dense and essential modules and try to establish the dense ideal structure of some incidence rings.

Dynamical Quantum Groups - The Super Story

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Let $\mathfrak{g}$ be a Lie algebra. The *classical Yang-Baxter equation* for $r = \sum_i a_i \otimes b_i \in \mathfrak{g} \otimes \mathfrak{g}$ is:

$$[r^{12}, r^{13}] + [r^{12}, r^{23}] + [r^{13}, r^{23}] = 0,$$

where the notation r^{12} stands for $\sum_i a_i \otimes b_i \otimes 1$, r^{13} stands for $\sum_i a_i \otimes 1 \otimes b_i$, and r^{23} stands for $\sum_i 1 \otimes a_i \otimes b_i$. A solution r to the classical Yang-Baxter equation is called an *r-matrix*. $r-$matrices and the classical Yang-Baxter equation are important concepts studied in integrable system theory.

A complete classification of nonskewsymmetric $r-$matrices exists in the case when $\mathfrak{g}$ is simple; see [1] and [2] for the original proofs by Belavin and Drinfeld, and [5] for a more pedagogical exposition. A similar construction, with natural modifications, works in the super case as well; see [6]. However, it turns out that this may not be easily modified into a full classification result; see [7] for an explicit construction and detailed study of a counterexample.

Solutions of the classical Yang-Baxter equation on a Lie algebra give us the semiclassical limits of quantizations on the associated Lie group. In [4], Etingof, Schedler and Schiffmann have explicitly constructed quantizations associated to all solutions coming from the Belavin-Drinfeld result. Their method in fact works for all dynamical $r-$matrices, i.e. the solutions of the more general dynamical Yang-Baxter equation. The quantized objects in this more general framework are the dynamical quantum groups. By now the theory of the classical and quantum dynamical Yang-Baxter equations and their solutions has many applications, in particular to integrable systems and representation theory. For a recent survey of results and such applications, one can refer to [3].

The purpose of this talk will be to present the beginnings of the super analog of the theory of dynamical quantum groups. We will provide a historical introduction to dynamical quantum groups and Lie superalgebras, and there will be interesting examples and various construction results, some of which may be found in [8].

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Higher Abel-Jacobi maps and elliptic functions

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We describe some new Hodge-theoretic invariants for detecting rational equivalence-classes of 0-cycles on a projective variety X. To keep the talk accessible and geometrically appealing, much of it will be spent working a "toy model" example where the variety is a product of two elliptic curves (and the cycle in the Albanese kernel). If time permits we will briefly explain brand-new results for exterior products of 0-cycles.

Orbits in the anti-invariant sublattice of the K3-lattice

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When a K3-surface X doubly-covers an Enriques surface, the covering transformation induces an involution on $H^2(X,\mathbb{Z})$. This cohomology group forms a lattice Λ under the cup-product (called the K3-lattice), and as such is isometric to $E_8^2 \oplus U^3$. Its anti-invariant sublattice is denoted by Λ^- and it is isometric to $E_8(2) \oplus U(2) \oplus U$. In this talk, we will determine the number of orbits of primitive cohomology classes in Λ^- under the action of its self-isometries. We will also try to derive certain geometric conclusions on curves on K3 surfaces and on divisors of the moduli space of Enriques surfaces. The work is joint with Prof. Sinan Sertöz.

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Absolute Irreducibility of Polynomials by the Polytope Method

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For any field F, a polynomial f in $F[x_1, x_2, ..., x_k]$ can be associated with a polytope, called its Newton polytope. If the polynomial f has integrally indecomposable Newton polytope, in the sense of Minkowski sum, then it is absolutely irreducible over F, i.e. irreducible over every algebraic extension of F.

In this talk, we present some results giving integrally indecomposable classes of polytopes. Consequently, we have some criteria giving infinitely many types of absolutely irreducible polynomials over arbitrary fields.

Algebraic Cycles and Mumford-Griffiths Invariants

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Let X be a compact Riemann surface. A zero-cycle is a formal sum $\xi := \sum_{j=1}^{N} n_j p_j$, where the n_j's are integers and the p_j's are points in X. Its degree is given by $\sum_{j=1}^{N} n_j$. If $\deg \xi = 0$, then ξ bounds a 1-chain ζ. If ξ also bounds ζ_0, then $\gamma := \zeta - \zeta_0$ is a 1-cycle (zero boundary), and integrals of holomorphic differentials (Abelian differentials) over γ are called periods. The story for Abelian integrals began with the works of Abel and Jacobi. A crowning achievement in the 19th century, due to Abel in his study of Abelian integrals, says that $\int_\zeta$ as an operator acting on the Abelian differentials on X, is a period iff ξ is the divisor of zeros minus poles of some rational (= meromorphic) function on X. Such a zero-cycle is called a principal divisor. The group of zero-cycles on X, modulo principal divisors, is called the Chow group of zero-cycles on X, and is denoted by $\mathrm{CH}_0(X)$. The degree map gives a surjection $\deg : \mathrm{CH}_0(X) \twoheadrightarrow \mathbb{Z}$; the kernel is given by a compact complex g-dimensional torus (jacobian), where g is the genus of X. If X is now a projective algebraic manifold of dimension $d \geq 1$ (a generalization of a compact Riemann surface), and $0 \leq k \leq d$, then one can still define the Chow group $\mathrm{CH}_k(X)$, involving k-dimensional algebraic cycles on X. Around the 1960's, and due to the seminal works of Griffiths (generalization of Abelian integrals) and Mumford (using Abelian differentials to investigate rational equivalence), the "story" for $\mathrm{CH}_k(X)$ took a nonclassical turn for $k < d-1$. It then eventually became clear to a number of mathematicians that $\mathrm{CH}_k(X) \otimes \mathbb{Q}$ is best understood in terms of a descending filtration $\mathrm{CH}_k(X) \otimes \mathbb{Q} = F^0 \supset F^1 \supset \cdots$, whose graded pieces Gr_F^ν are conjecturally described in terms of some (motivic) extension datum. Indeed, this conjectural filtration has been a focus of attention for the past 25 years. In this talk, I'll discuss some recent developments, by introducing some arithmetic Hodge theoretic invariants that can be used to detect interesting classes in Gr_F^ν. This is based on joint work with Shuji Saito.

Quotient of PQ-hyperstructures

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The theory of hyperstructures has been introduced by Marty in 1934 during the 8^{th} congress of the Scandinavian Mathematicians [1]. Marty introduced the notion of a hypergroup and then many researchers have been worked on this new topic of modern algebra and developed it.

In this talk we deal with the class of the H_v-hyperstructures which was first introduced by T. Vougiouklis in the fourth AHA Congress, Xanthi (1990), where the classical axioms are replaced by weaker ones. The weak axioms are the ones where the non-empty intersections replaces equality. Moreover one can use the so called P-hyperoperations which are defined on a given structure by using any non-empty subset [2], [3], [4].

In this lecture first we introduce PQ-hyperoperations which are generalization of P-hyperoperations induced by subsets of a ring R which is the ground ring of a module M. In this respect, some theorems about PQ-hyperoperations are proved. Also, the homomorphisms between PQ-H_v-modules are studied.

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A Characterization of Alternating Groups

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Let I be a set of arbitrary cardinality. Denote by $A(I)$ the alternating group on I, i.e. the group of all almost trivial even permutations on I. If X is the set of all

3-*cycles* $(i,j,k) \in A(I), i,j,k \in I, i \neq j \neq k \neq i$, then X is a conjugacy class in $A(I)$, $\langle X \rangle = A(I)$ and, for every non-commuting $x, y \in X$, $\langle x, y \rangle$ is isomorphic to A_4 or A_5 where A_n for a natural number n denotes the alternating group of degree n.

One of goals of this talk is a characterization of $A(I)$ by the above properties of class X.

Theorem. *Let G be a group generated by a conjugacy class X of elements of order 3 such that every two non-commuting members of X generate a group isomorphic to A_4 or A_5. Then either $G = T\langle x \rangle$ where T is an elementary abelian normal subgroup, $x \in X$ and $C_T(x) = 1$, or there exists a set I of cardinality at least 5 such that $G \simeq A(I)$. In particular, G is locally finite.*

For finite groups, this theorem is, essentially, a particular case of the main result in [1].

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The inductive closure of the proper class of supplements in abelian groups

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Let A be a submodule of a module B. A is said to be a *complement* in B if there exits a submodule $K \leq B$ such that $K \cap A = 0$ and A is *maximal* with respect to this property. A is said to be a *supplement* in B if there exits a submodule $K \leq B$ such that $B = K + A$ and A is *minimal* with respect to this property. See [2] and [12, §41].

We deal with *complements* (*closed submodules*) and *supplements* in unital R-modules for an associative ring R with unity using *relative* homological algebra via the two *proper classes* of short exact sequences of R-modules and R-module homomorphisms, $\mathcal{C}ompl_{R\text{-}\mathcal{M}od}$ and $\mathcal{S}uppl_{R\text{-}\mathcal{M}od}$, and related other proper classes like $\mathcal{N}eat_{R\text{-}\mathcal{M}od}$. $\mathcal{C}ompl_{R\text{-}\mathcal{M}od}$ [$\mathcal{S}uppl_{R\text{-}\mathcal{M}od}$] consists of all short exact sequences

$$0 \longrightarrow A \xrightarrow{f} B \xrightarrow{g} C \longrightarrow 0$$

of R-modules and R-module homomorphisms such that $\mathrm{Im}(f)$ is a complement [resp. supplement] in B. [4, 5] give more general definitions for proper classes of complements and supplements related to another given proper class. $\mathcal{N}eat_{R\text{-}\mathcal{M}od}$ consists of all short exact sequences of R-modules and R-module homomorphisms with respect to which every simple R-module is projective (following [10, 9.6 in §9] and [11]). For terminology and notation in *proper classes*, we shall follow [9] (see also [6, Ch. 12, §4] or [10]).

A proper class $\mathcal{P}$ is said to be *inductively closed* if for every direct system $\{\mathbb{E}_i (i \in I); \pi_i^j (i \leq j)\}$ in $\mathcal{P}$, the direct limit $\mathbb{E} = \varinjlim \mathbb{E}_i$ is also in $\mathcal{P}$ ([3] and [9, §8]). The smallest inductively closed proper class containing a proper class $\mathcal{P}$ is called the *inductive closure* of $\mathcal{P}$.

We shall consider the case in abelian groups, i.e. $R = \mathbb{Z}$, the ring of integers.

The proper class $\mathcal{C}ompl_{\mathbb{Z}\text{-}\mathcal{M}od} = \mathcal{N}eat_{\mathbb{Z}\text{-}\mathcal{M}od}$ is *projectively generated*, *flatly generated* and *injectively generated* by simple abelian groups $\mathbb{Z}/p\mathbb{Z}$, p prime number:

$$\begin{aligned} \mathcal{C}ompl_{\mathbb{Z}\text{-}\mathcal{M}od} &= \mathcal{N}eat_{\mathbb{Z}\text{-}\mathcal{M}od} = \pi^{-1}(\{\mathbb{Z}/p\mathbb{Z} | p \text{ prime}\}) \\ &= \tau^{-1}(\{\mathbb{Z}/p\mathbb{Z} | p \text{ prime}\}) = \iota^{-1}(\{\mathbb{Z}/p\mathbb{Z} | p \text{ prime}\}). \end{aligned}$$

The inductive closure of the proper class $\mathcal{S}uppl_{\mathbb{Z}\text{-}\mathcal{M}od}$ is flatly generated by all simple abelian groups, so it is equal to $\mathcal{C}ompl_{\mathbb{Z}\text{-}\mathcal{M}od} = \mathcal{N}eat_{\mathbb{Z}\text{-}\mathcal{M}od}$.

To every proper class $\mathcal{P}$, we have a relative $\mathrm{Ext}_{\mathcal{P}}$ functor and for the proper class $\mathcal{S}uppl_{\mathbb{Z}\text{-}\mathcal{M}od}$, unlike $\mathcal{C}ompl_{\mathbb{Z}\text{-}\mathcal{M}od}$, this functor behaves badly in the sense that the functor $\mathrm{Ext}_{\mathcal{S}uppl_{\mathbb{Z}\text{-}\mathcal{M}od}}$ is not *factorizable* as

$$\mathbb{Z}\text{-}\mathcal{M}od \times \mathbb{Z}\text{-}\mathcal{M}od \xrightarrow{\mathrm{Ext}_{\mathbb{Z}}} \mathcal{A}b \xrightarrow{H} \mathcal{A}b$$

for any functor $H : \mathcal{A}b \longrightarrow \mathcal{A}b$ on the category $\mathbb{Z}\text{-}\mathcal{M}od = \mathcal{A}b$ of abelian groups.

Some of these results can be generalized to modules over Dedekind domains using the results in [8].

Acknowledgements.
These results are from [1] with co-author Rafail Alizade (İzmir Institute of Technology, Turkey; email: `rafailalizade@iyte.edu.tr`). He is my Ph. D. Thesis advisor whom I wish to express my thanks once more (see [7]).
I would like to express my gratitude to TÜBİTAK (The Scientific and Technical Research Council of Turkey) for its support during my Ph. D. research. Besides, TÜBİTAK has given services like in obtaining articles easily from various libraries via ULAKBİM which has been so useful.

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The Poincaré series of the module of derivations of some monomial rings

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Let R be a graded k-algebra and M be a finitely generated graded R-module. The formal power series $\sum_i \dim_k \operatorname{Tor}_i^R(k, M)z^i$ is called the Poincaré series and it is denoted by $P_M^R(z)$. We show, as a particular case of a more general result that we prove, that the Poincaré series of the module of derivations of monomial rings is always rational and we determine it in a lot of cases.

MSC: 13D02; 13D07

On the motive of an algebraic surface

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In the lecture we intend to discuss the Chow motives which one can associate to an algebraic surface and to outline some of their properties.

Clean Endomorphism Rings

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A ring is called clean if every element is the sum of an idempotent and a unit. Every clean ring is an exchange ring, and every semiperfect ring and every unit regular ring is clean.. Recently a lot of work has been done on the question when the endomorphism ring of a module is clean. The talk will survey this subject and present some recent results.

Cojective Modules in the class of $\mathcal{B}(M, X)$

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Extending modules, lifting modules and other related concepts as interesting generalizations of concepts of projectivity and injectivity have been studied extensively in recent years by many authors .
L. Permouth, K. Oshiro and S.T. Rizvi define a family $\mathcal{A}(X, M)$ of submodules of M as follows:

$\mathcal{A}(X, M) = \{A \leq M \mid \exists Y \leq X, \exists f \in Hom(Y, M), f(Y) \leq_e M\}$

and investigate $\mathcal{A}(X, M)$-extending modules. Dually D. Keskin and A. Harmancı define a family as follows and investigate $\mathcal{B}(M, X)$-lifting modules

$\mathcal{B}(M, X) = \{A \leq M \mid \exists Y \leq X, \exists f \in Hom(M, X/Y), Ker f/A \ll M/A\}$

S. H. Mohamed and B. J. Müller defined cojective modules as a generalization of projectivity. A is *B–cojective* if, for any homomorphism $\psi : A \to X$ and any epimorphism $\pi : B \to X$, there exist decompositions $A = A_1 \oplus A_2$, $B = B_1 \oplus B_2$,

a homomorphism $\psi_1 : A_1 \to B_1$ and an epimorphism $\psi_2 : B_2 \to A_2$ such that $\pi\psi_1 = \psi \mid_{A_1}$ and $\psi\psi_2 = \pi \mid_{B_2}$.
In this note I give some characterizations of lifting modules in terms of cojective modules and the class of $\mathcal{B}(M, X)$.
Result1: let $M = M_1 \oplus M_2$ be an X–amply supplemented module with the finite internal exchange property. Then for every decomposition of $M = M_i \oplus M_j$, M_i is $\mathcal{B}(M_j, X)$–cojective for $i \neq j$, M_1 and M_2 are X–lifting if and only if M is X–lifting.
Result2: Let M_1 and M_2 be indecomposable X–lifting modules and let $M = M_1 \oplus M_2$ be an X–amply supplemented module. If one of the following conditions holds, then M is X–lifting.
(1) M_1 is small–$\mathcal{B}(M_2, X)$–cojective and every X–supplement submodule N of M such that $M = N + M_1$ is a direct summand.
(2) M_1 is small–$\mathcal{B}(M_2, X)$–cojective, M_2 is small–$\mathcal{B}(M_1, X)$–cojective and M_1 is pseudo–$\mathcal{B}(M_2, X)$–cojective.
(3) M_1 is small–$\mathcal{B}(M_2, X)$–cojective, M_2 is small–$\mathcal{B}(M_1, X)$–cojective and M_2 is pseudo–$\mathcal{B}(M_1, X)$–cojective.
(4) M_2 is $\mathcal{B}(M_1, X)$–cojective and M_1 is small–$\mathcal{B}(M_2, X)$–cojective.
(5) M_1 is simple and small–$\mathcal{B}(M_2, X)$–cojective.
(6) M_1 is simple and almost $\mathcal{B}(M_2, X)$–projective.

This is a joint work with Derya Keskin Tütüncü (Hacettepe University).

Relative Flux Homomorphism in Symplectic Geometry

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In this work, we define a relative version of the flux homomorphism, introduced by Calabi in 1969, for a symplectic manifold ([1]). We use it to study (the universal cover of) the group of symplectomorphisms of a symplectic manifold leaving a Lagrangian submanifold invariant, mainly following [2]. We also show that some quotients of the universal covering of the group of symplectomorphisms are stable under symplectic reduction.

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Some recent results on codes and curves

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We give a survey of some of our recent results on the improvements of the bounds of Weil-type exponential sums over Galois rings and their applications in the construction of codes and sequences.

This is a report on some joint works with San Ling.

Semiperfect Modules With Respect to a Fully Invariant Submodules

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A ring R with identity is called a *semiperfect ring* if for every left (or right) ideal A of R, there exists an idempotent $e^2 = e \in R$ such that $A = Re \oplus B$ and $B \subseteq J$ (J is the Jacobson radical of R). Note that left or right Artinian rings are semiperfect.

Recently this notion is generalized by Yousif and Zhou to I–semiperfect rings R by considering any ideal I of R instead of the Jacobson radical in the definition of semiperfect rings. After that it is usual to consider the module version of this definition. Let M be a left R–module. We study on the category $\sigma[M]$ which is a more general category of all left R–modules. $\sigma[M]$ consists of left R–modules N such that $N \hookrightarrow M^{(\Lambda)}/K$ for some index set Λ and a submodule K of $M^{(\Lambda)}$. Recall that a submodule U of a module M is called *fully invariant* if $f(U) \subseteq U$ for every endomorphism $f : M \to M$. Note that any ideal of a ring R is fully invariant as a module over R.

Let U be a fully invariant submodule of $N \in \sigma[M]$. We call $N \in \sigma[M]$ a U–*semiperfect module* if for any submodule K of N, there exists a decomposition $K = A \oplus B$ such that A is a projective direct summand of N in $\sigma[M]$ and $B \subseteq U$.

We investigate conditions equivalent to being U–semiperfect focusing on certain fully invariant submodules such as $Z_M(N)$ (the M–singular submodule), $Soc(N)$ (socle = sum of simple submodules of N) and $\delta_M(N)$. Results are applied to characterize Quasi-Frobenius (QF) rings (with $J^2 = 0$) and semisimple rings.

This is joint work with Mustafa Alkan (Akdeniz University)

Homology of real algebraic varieties and morphism to sphere

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In this talk, we consider when a smooth map from a nonsingular real algebraic variety X into the standard sphere S^n can be homotoped to a regular map. The cases S^1, S^2 and S^4 were studied in [1]. For the general case, we have the following result: Let $f : X^{2n} \to S^{2n}$ be a smooth map, where X^{2n} is a compact connected nonsingular orientable real algebraic variety. If there is a cohomology class $u \in H^2_{\mathbb{C}-alg}(X, \mathbb{Z})$ such that $u^n = f^*(\alpha)$, where $\alpha \in H^{2n}(S^{2n}, \mathbb{Z})$ is a generator, then f is homotopic to a regular map.

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Differential fields of arbitrary characteristic

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Saunders Mac Lane died this year at 95. In the 1930s, he introduced and studied some of the ideas that I shall discuss.

In a vector-space over a field K, one has the notion of taking the linear span of a set:

$$A \longmapsto \langle A\rangle^K = \sum_{v\in A} Kv \qquad \text{(finite sums).}$$

In a field-extension L/K, one can take (relative) algebraic closures over K:

$$\mathrm{cl}^0_K : A \longmapsto K(A)^{\mathrm{alg}} \cap L.$$

As may be observed in an algebra class, these operations have the same formal properties, yielding in each case notions of **independence** (linear or algebraic) and **basis** (linear or transcendence-).

If characteristic of the field-extension is a prime p, then there is an other closure-operator,

$$\mathrm{cl}^p_K : A \longmapsto L^p K(A),$$

yielding the notion of a **relative p-basis** [1]. The operators cl^0 and cl^p have a uniform definition in terms of the **universal derivation** d_K of L/K:

Let $\mathrm{Der}(L/K)$ be the set of derivations D from L to itself that are trivial on K (so D is a K-linear endomorphism satisfying $D(x \cdot y) = y \cdot Dx + x \cdot Dy$.) Let d_K be the K-linear map

$$x \longmapsto (D \to Dx) : L \longrightarrow \mathrm{Der}(L/K)^*$$

Then

$$x \in \mathrm{cl}^{\mathrm{char}\,K}_K(A) \iff \mathrm{d}_K\, x \in \langle \mathrm{d}_K\, a : a \in A\rangle^L.$$

Such considerations lead to a uniform treatment of differential fields of arbitrary characteristic—in particular, a uniform characterization of the *existentially closed* differential fields.

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Bezoutians, Euclidean algorithm, and orthogonal polynomials

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This is a report on a joint work [4] with Alain Lascoux. Suppose that f, φ is an arbitrary pair of univariate polynomials. We set (after Bézout) for variables x, y,

$$\mathrm{Bez}(f,\varphi) := \frac{f(x)\varphi(y) - f(y)\varphi(x)}{x-y},$$

and call this bivariate polynomial the *Bezoutian* (of f and φ). There has recently been a revival of interest in Bezoutians because of their importance in many diverse fields of numerical and symbolical computing as well as in control theory.

We give in [4] the following formula (missed by classics):

$$\mathrm{Bez}(f,\varphi) = p_0\varphi(x)\varphi(y) + \sum_{i=1}^{n} p_i \mathcal{R}_i(x)\mathcal{R}_i(y)\,, \qquad (*)$$

where $p_i = \frac{\mathcal{Q}_i(x)-\mathcal{Q}_i(y)}{x-y}$, and $\mathcal{R}_i$ and $\mathcal{Q}_i$ come from the Euclidean algorithm (performed with nonstandard signs):

$$f = \mathcal{Q}_0\varphi - \mathcal{R}_1,\quad \varphi = \mathcal{Q}_1\mathcal{R}_1 - \mathcal{R}_2,\quad \mathcal{R}_1 = \mathcal{Q}_2\mathcal{R}_2 - \mathcal{R}_3\ ,\ldots$$

$$\ldots, \mathcal{R}_{n-2} = \mathcal{Q}_{n-1}\mathcal{R}_{n-1} - \mathcal{R}_n,\quad \mathcal{R}_{n-1} = \mathcal{Q}_n\mathcal{R}_n\,.$$

We say that a pair (f,φ) is *general* if the Euclidean quotients $\mathcal{Q}_i$ are of degree 1 for $i = 1,\ldots,n$. From now on, we assume that (f,φ) is a general pair of monic polynomials of degrees $(n+1, n)$ with alphabets of roots $\mathbb{A}$ and $\mathbb{B}$. Using some Schur function formulas from [2] and [3], we deduce from the identity $(*)$ the following one:

$$\mathrm{Bez}(f,\varphi) = \varphi(x)\varphi(y) + \sum_{i=1}^{n}(-1)^i \frac{S_{1^{n-i};(i+1)^i}(\mathbb{B}-x;\mathbb{B}-\mathbb{A})\,S_{1^{n-i};(i+1)^i}(\mathbb{B}-y;\mathbb{B}-\mathbb{A})}{S_{i^{i-1}}(\mathbb{B}-\mathbb{A})\,S_{(i+1)^i}(\mathbb{B}-\mathbb{A})},$$

and the following congruence modulo $(f(x), f(y))$:

$$\mathrm{Bez}(f,\varphi) \equiv \varphi(x)\varphi(y)\Big(1 + \sum_{i=1}^{n}(-1)^i \frac{S_{i^i}(\mathbb{A}-\mathbb{B}-x)\,S_{i^i}(\mathbb{A}-\mathbb{B}-y)}{S_{(i-1)^i}(\mathbb{A}-\mathbb{B})\,S_{i^{i+1}}(\mathbb{A}-\mathbb{B})}\Big)\,. \qquad (\dagger)$$

Consider now the functional

$$\mu : g(x) \mapsto \sum_{a\in\mathbb{A}} g(a)\frac{\varphi(a)}{\prod_{b\neq a}(a-b)}\,.$$

The functional μ is an incarnation of the Lagrange interpolation in the points $a \in \mathbb{A}$. It is characterized by the fact that it sends each x^i, $i \in \mathbb{N}$, onto the complete function $S_i(\mathbb{A}-\mathbb{B})$. The Schur polynomials $P_i = S_{i^i}(\mathbb{A}-\mathbb{B}-x)$ for $i = 0, 1, \ldots, n$, form a unique (up to normalization) orthogonal basis with respect to the functional μ, of polynomials of respective degrees $0, 1, \ldots, n$. We rewrite the congruence (†), with the help of the *Christoffel-Darboux kernel*, $K(x,y) = \sum_{i=0}^{n} P_i(x)P_i(y)/\mu\big(P_i(x)^2\big)$, as $\mathrm{Bez}(f,\varphi) \equiv \varphi(x)\varphi(y)\,K(x,y)$. This suggests that $\mathrm{Bez}(f,\varphi)$ – similarly to $K(x,y)$ – should have a *reproducing* property. Indeed, the following property is proved in [4]: for a polynomial $g(x)$,

$$\mu\big(g(x)\ \mathrm{Bez}(f,\varphi)\big) \equiv \varphi(y)^2\, g(y) \mod f(y)\,.$$

In [4], we also interpret, in the language of orthogonal polynomials various properties (discovered first by Sylvester [5] and Brioschi [1]) of the numerators and denominators $\mathcal{D}_i$ of the successive convergents of the continued fraction

$$\frac{\varphi}{f} = \cfrac{1}{\mathcal{Q}_0 - \cfrac{1}{\mathcal{Q}_1 - \cfrac{1}{\ddots\, \cfrac{1}{\mathcal{Q}_n}}}}\ .$$

For example,

$$\sum_{a\in\mathbb{A}} \mathcal{D}_i(a)\mathcal{D}_j(a)\frac{\varphi(a)}{\prod_{b\neq a}(a-b)} = 0\,.$$

We have similar results for a general pair of two univariate polynomials of the same degree (cf. [4]).

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Modules of finite torsion-free rank over a discrete valuation domain

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Let R be a discrete valuation domain with $Q \neq R$ its quotient field. Let pR denote the unique maximal ideal of R. Non-zero finite rank pure submodules of the completion $\hat{R}$ of R are called purely indecomposable modules and possess interesting properties and behave like the additive group of rational numbers.

Theorem 1. *A reduced finite rank torsion-free R-module A is purely indecomposable $\Leftrightarrow$ every pure submodule of A is indecomposable $\Leftrightarrow$ $A/pA \cong R/pR$ $\Leftrightarrow$ every proper torsion-free homomorphic image of A is divisible $\Leftrightarrow$ A has a pure cyclic submodule B which is dense in A under the p-adic topology, that is, A/B is divisible.*

An pure exact sequence of R-modules $0 \to A \to B \overset{\eta}{\to} C \to 0$ is said to *pi-balanced* if, for any purely indecomposable module X and any homomorphism $\alpha : X \to C$, there is a homomorphism $\beta : X \to B$ such that $\eta\beta = \alpha$. Pullbacks and pushouts of pi-balaced exact sequences are again pi-balanced and the pi-balanced exact sequences form a proper class in the sense of MacLane. So the inequivalent pi-balanced extensions of A by C form a subgroup $PBext^*(C, A)$of the group $Ext_R(C, A)$ of all the extensions of A by C.

Theorem 2. *An R-module C satisfies $Bext^*(C, A) = 0$ for all R-modules A if and only if $C = D \oplus E \oplus F$, where D is a torsion divisible module, E is a direct sum of cyclic modules and F is a direct sum of purely indecomposable modules.*

Our goal is to describe R-modules M which have the property that $PBext^*(M, T) = 0$ for all torsion modules T. Finite rank torsion-free abelian groups which satisfy a similar splitting condition of $Bext(M, T) = 0$ have turned out to be an important class of groups called Butler groups possessing many interesting properties. In view of this, we consider mixed modules M of finite torsion-free rank for which $Bext^*(M, T) = 0$ for all torsion modules T and call them *pi-Butler modules.*

Theorem 3. *For a torsion R-module M, $Bext^*(M, T) = 0$ if and only if $M = D \oplus S$, where D is torsion divisible and S is a direct sum of torsion cyclic modules.*

Next we show that if M is a mixed module of finite torsion-free rank with $PBext^*(M, T) = 0$ for torsion t, then, for any full free submodule F, M/F also satisfies the same proerty. Using this we get:

Theorem 4. *Let M be a mixed module of finite torsion-free rank, then $PBext(M, T) = 0$, for all torsion modules T if and only if $M = T^* \oplus D \oplus N$, where T^* is a direct sum of torsion cyclic module, D is torsion divisible and $N = X_1 + \cdots + X_k$ where each X_i is a rank-2 purely indecomposable module.*

The next set of results investigates the conditions under which the finite rank torsion-free module N above can be determined by ismorphism inavriants.

Description of the superinvolutions for M(2)

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Concerning the matrix algebra $M_n(K)$ over a field K of characteristic zero due to the identities satisfied the important involutions (i.e. antiautomorphisms of order two) are the transpose involution (t) and the symplectic one (s). The latter is defined for n even as $*$ given by

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}^* = \begin{pmatrix} D^t & -B^t \\ -C^t & A^t \end{pmatrix},$$

for $A, B, C, D \in M_{\frac{n}{2}}(K)$.

Using the notation $(R, *)$ for an algebra R with involution $*$ we get $R = R^+ \oplus R^-$, where $R^+ = \{r \in R : r^* = r\}$ is the symmetric part and $R^- = \{r \in R : r^* = -r\}$ is the skew-symmetric part of the considered algebra as $r = \frac{r+r^*}{2} + \frac{r-r^*}{2}$.

One could investigate the $*$-identities either in symmetric or in skew-symmetric variables. Many results were obtained for some special polynomials called Bergman type polynomials due to V. Drensky, M. Racine, Ts. Rashkova. These results are the impulse to generalize the problem for other algebras as well. These algebras are defined in the talk.

Suppose that $G =< \varphi >$ is the multiplicative group of order 2. Then $\frac{1+\varphi}{2}$ and $\frac{1-\varphi}{2}$ are the minimal idempotents of the group algebra FG. The free algebra with G-action is freely generated by the elements $x_i + x_i^\varphi$and $x_i - x_i^\varphi, i = 1, \ldots$. For convenience, for every i, let us write $x_i + x_i^\varphi = y_i$ and $x_i - x_i^\varphi = z_i$. Then $F < X \mid G >= F < Y, Z >$ is the free associative algebra on the two sets $Y = \{y_1, y_2, \ldots\}$ and $Z = \{z_1, z_2, \ldots\}$.

When φ is an automorphism of order 2, then $F < Y, Z >$ has a structure of Z_2-graded algebra (or superalgebra) where the variables from Y have homogeneous degree 0 and the variables from Z have homogeneous degree 1. The algebra $F < Y, Z >$ is the free superalgebra on Y and Z.

In case φ is an involution, we write $\varphi = *$ and $F < Y, Z >$ has an induced structure of algebra with involution where the variables from Y are symmetric and the variables from Z are skew-symmetric.

Any associative superalgebra is a Z_2-graded K-algebra A such that

$$A = A_0 \oplus A_1,\ A_\alpha A_\beta \leq A_{\alpha+\beta}\, (\alpha, \beta \in Z_2).$$

An example of a superalgebra is the algebra of square matrices of order $r+s$ whose grading is determined in the following way:

$$M(r \mid s)_0 = \{\begin{bmatrix} A & 0 \\ 0 & D \end{bmatrix} \mid A \in M_r(K), D \in M_s(K)\},$$
$$M(r \mid s)_1 = \{\begin{bmatrix} 0 & B \\ C & 0 \end{bmatrix} \mid B \in M_{r,s}(K), C \in M_{s,r}(K)\}.$$

We denote this algebra as $M(n)$ for $r = s = n$.

Let A be an associative superalgebra. A superinvolution on A is a Z_2-graded linear map $* : A \to A$ such that, for all $a, b \in A, (a^*)^* = a$ and $(ab)^* = (-1)^{\bar{a}\bar{b}} b^* a^*$, where $\bar{x}$ means the parity of $x; \bar{x} = i$ if $x \in A_i, i = 0, 1$.

Examples of superinvolutions are

the orthosymplectic superinvolution *osp*, defined by

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{osp} = \begin{bmatrix} H & 0 \\ 0 & K \end{bmatrix}^{-1} \begin{bmatrix} A & -B \\ C & D \end{bmatrix}^{t} \begin{bmatrix} H & 0 \\ 0 & K \end{bmatrix},$$

where H is a symmetric matrix and K is a skew-symmetric one, both invertible and

the transposition superinvolution *trp*, defined by

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{trp} = \begin{bmatrix} D^t & -B^t \\ C^t & A^t \end{bmatrix}.$$

There is a result of Ambrozi and Shestakov [1, Theorem 3.2] describing all superinvolutions for $M(1)$.

In the paper we classify the superinvolutions for $M(2)$. Defining the even and the odd idempotents of $M(2)$ we consider all possible cases as every superinvolution is an involution on the even part. Standard but a lot matrix calculations give the final result. The following theorems are proved:

Theorem 1. *Let $M_2(K)$ has an involution $\tilde{}$. Then the formula*

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^* = \begin{pmatrix} \widetilde{d} & -\widetilde{b} \\ \widetilde{c} & \widetilde{a} \end{pmatrix},$$

defines a superinvolution on the simple superalgebra $M(2)$.

The possibilities for $\tilde{}$ are $\tilde{} = t, p \circ t, s, p \circ s$, where p is the parity automorphism.

All involutions for $M(2)$ are of type $p \circ *$.

Theorem 2. *Let $(A = M(2), * \mid_A)$ is not simple. Then in $A = A_0 \oplus A_1$ one $(A_i, * \mid_{A_i})$ is of orthogonal type and the other of symplectic type. The possibilities are either orthogonal type $t, p \circ t, t_1$ (t_1 being the transpose to the second diagonal) and $p \circ t_1$ and symplectic type s or vice versa.*

All involutions for $M(2)$ are of type $p \circ *$.

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Artinian Rings and Modules

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Let R be a ring. An R-module M satisfies the (H)-condition if the annihilator of M in R is the annihilator of a finite subset of M. It is well known that if R is a right Artinian ring then every right R-module satisfies the (H)-condition. However, for any commutative ring R, every finitely generated R-module satisfies the (H)-condition. In general, it turns out that if every countably generated right R-module satisfies the (H)-condition then R is a right Artinian ring. This result and various other results concerning Artinian rings and modules will be presented.

Zero cycles and complete intersection points on affine varieties

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Let $X = \operatorname{Spec} A$ be an irreducible affine variety of dimension d over an algebraically closed field k, so that the coordinate ring A of algebraic functions on X is a finitely generated k-algebra which is an integral domain of Krull dimension d. A *complete intersection point* of X is a point $x \in X$ such that the maximal ideal $\mathfrak{M}_x \subset A$ of functions vanishing at x is generated by d elements $f_1, \ldots, f_d \in \mathfrak{M}_x$. Geometrically, this means that $x \in X$ is a non-singular point, and the hypersurfaces $H_i = \{y \in X | f_i(y) = 0\}$ satisfy $H_1 \cap \cdots \cap H_d = \{x\}$, and the H_i intersect transversally near x.

We are interested in characterizing varieties $X = \operatorname{Spec} A$ such that all non-singular points $x \in X$ are complete intersections. This problem turns out to have different flavours, depending on the ground field k, and is related to interesting conjectures in the theory of algebraic cycles, and thereby to algebraic K-theory. In this talk, I will give an introduction to this topic, dwelling in particular on some recent results based on the paper [3]. Some survey articles giving background, detailed references, and explaining the connections with commutative algebra, are [2] and [1].

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Extended Hecke Groups and Their Some Normal Subgroups

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This is joint work with Sebahattin İkikardes and Özden Koruoğlu.

In [1], Erich Hecke introduced the groups $H(\lambda)$ generated by two linear fractional transformations

$$T(z) = -\frac{1}{z} \qquad \text{and} \qquad S(z) = -\frac{1}{z+\lambda},$$

where λ is a fixed positive real number. E. Hecke showed that H(λ) is Fuchsian if and only if $\lambda = \lambda_q = 2\cos\frac{\pi}{q}$, where q is an integer ≥ 3, or $\lambda \geq 2$ is real. In these two cases $H(\lambda)$ is called a Hecke group. We consider the former case. Then the Hecke group $H(\lambda_q)$ is the discrete subgroup of $PSL(2,\mathbb{R})$ generated by T and S, and it has a presentation

$$H(\lambda_q) =< T, S \mid T^2 = S^q = I >\cong C_2 * C_q.$$

The extended Hecke groups, denoted by $\overline{H}(\lambda_q)$, have been defined in [3] and [4] by adding the reflection $R(z) = 1/\overline{z}$ to the generators of the Hecke group $H(\lambda_q)$. Thus the extended Hecke group $\overline{H}(\lambda_q)$ has the presentation

$$\overline{H}(\lambda_q) =< T, S, R \mid T^2 = S^q = R^2 = (TR)^2 = (RS)^2 = I > .$$

Here, firstly, we give the abstract group structure of the extended Hecke groups $\overline{H}(\lambda_q)$. Then, we find the abstract group structures and generators of the power subgroups $\overline{H}^m(\lambda_q)$, $m \in \mathbb{Z}^+$, and commutator subgroups of $\overline{H}(\lambda_q)$. This we achieve by applying the Reidemeister-Schreier method of combinatorial group theory. Also, we determine the relations between power subgroups and commutator subgroups of $\overline{H}(\lambda_q)$. Finally, we investigate free normal subgroups of finite index in the extended Hecke groups $\overline{H}(\lambda_q)$.

(This talk is based on the references [5], [6], [7]).

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Formal Languages and Groups

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One of the classical results in group theory is the unsolvability of the word problem for finitely presented groups [1, 5]; this says that there are finite presentations such that there is no algorithm to decide whether or not a word in the generators represents the identity element of the group defined by the presentation. An alternative way of describing this situation is as follows: there are finitely presented groups G such that, if we consider the set W of all words representing the identity element of G, then there is no algorithm for determining membership of W. There is also an elegant result of Boone and Higman [2] describing which finitely generated groups have a solvable word problem.

One natural question that arises from this is the following: if we take some restricted model of computation, one can ask which groups have a word problem which is decidable within that model. This approach relates to *formal language theory* where a class of languages is described in terms of (for example) a type of abstract machine that determines membership of such a language. (A *language* in this setting is just a set of words.)

The purpose of this talk is to survey some of what is known in this field; see [3, 4, 7] for example. In particular, we shall report on some recent results [6] concerning groups whose word problem is the complement of a context-free language. We will not assume any prior knowledge of formal language theory and only some limited notions from the theory of groups.

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Property (τ) and the classification problem for the torsion-free abelian groups of finite rank

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In 1937, Baer solved the classification problem for the torsion-free abelian groups of rank 1. Since then, despite the efforts of such mathematicians as Kurosh and Malcev, no satisfactory solution has been found for the classification problem for the torsion-free abelian groups of rank $n \geq 2$. So it is natural to ask whether the classification problem is genuinely difficult for the groups of rank $n \geq 2$. In this talk, I will explain how this question can be partially answered, using Zimmer's superrigidity theorems for actions of irreducible lattices in products of real and p-adic Lie groups.

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Absolutely Supplement Modules

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An R-module will mean left R-module where R be an associative ring with identity. A submodule N of a module M is a supplement (respectively complement) of $K \leq M$ if $N + K = M \quad (N \cap K = 0)$ and N is minimal (respectively maximal) with respect to this property [1], [3].

Definition. A module M is called an absolutely supplement module if it is a supplement in every module containing it.

M is an absolutely supplement module if and only if M is a supplement in its injective envelope $E(M)$.

Some properties of absolutely supplement modules are as follows: every finite direct sum of absolutely supplement modules is an absolutely supplement module; every supplement submodule of an absolutely supplement module is absolutely supplement.

Proposition 1. *For a submodule N of a module M if N and M/N are absolutely supplement then M is absolutely supplement.*

A module M is a complement in every module containing it if and only if M is a complement in its injective envelope if and only if M is injective.

Definition. M is called an absolutely co-supplement (absolutely co-complement) if for every module X and T where $T \leq X$ with $X/T \cong M$, T is a supplement (complement) in X.

Proposition 2. *M is an absolutely co-supplement (absolutely co-complement) module if and only if there exists a projective (respectively injective) module P with $N \leq P$ and $P/N \cong M$ such that N is a supplement (respectively complement) submodule of P.*

If R is a Dedekind domain with $Rad\ R = 0$ then absolutely co-supplements are only projective modules. If R is a Dedekind domain then absolutely co-complements are only torsion-free $R-$modules [2].

Joint work with: Rafail Alizade; İzmir Institute of Technology.

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On Bergman's property for the automorphism groups of relatively free groups

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A group G is said to have *finite width* relative to a generating set X if there is a natural number k such that every element of G can be expressed as a product of at most k elements of $X^{\pm 1}$. A group with *Bergman's property* is a group which has finite width relative to any generating set.

The property is named after George Bergman who proved recently that it is satisfied by all infinite symmetric groups [1]. This result attracted a considerable attention and soon another examples of groups with Bergman's property have been found.

We shall discuss our partial answer to one of the questions from [1]: does the automorphism group of a free group of infinite rank have Bergman's property? We shall also give a sketch of the proof of the fact that the automorphism group of any free nilpotent group of infinite rank has Bergman's property.

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Calabi-Yau Orbifolds

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This is a report on an on-going project by the author. Let M be a Calabi-Yau manifold with a faithful action by a finite group G such that $M/G \simeq \mathbb{P}^n$. We study such pairs (M, G) and give a complete list in case G is abelian, We also exhibit some pairs with a non-abelian G.

A basic example of the pair (M, G) is the following: Let

$$S : \{[z_0 : \cdots : z_n] \in \mathbb{P}^n \mid P(z_0, \ldots, z_n) = 0\}$$

be a smooth hypersurface in $\mathbb{P}^n$ defined by a homogeneous polynomial P of degree $n+2$. Then the hypersurface in $\mathbb{P}^{n+1}$ defined as

$$M : \{[z_0 : \cdots : z_{n+1}] \in \mathbb{P}^{n+1} \mid z_{n+1}^{n+2} = P(z_0, \ldots, z_n)\}$$

is smooth of degree $n+2$, which implies that M is a Calabi-Yau variety of dimension n. In dimension two, M is a smooth quartic surface and in dimension three M is a smooth quintic threefold. Let ω be an $n+2$nd root of unity. Then the cyclic group $G := \mathbb{Z}/(n+2)$ acts on M, the action of $i \in G$ is given by

$$[z_0 : \cdots : z_{n+1}] \in M \to [z_0 : \cdots : \omega^i z_{n+1}] \in M$$

Consider the projection

$$\varphi : [z_0 : \cdots : z_{n+1}] \in \mathbb{P}^{n+1} \to [z_0 : \cdots : z_n] \in \mathbb{P}^n,$$

its restriction to M is precisely the quotient map $M \to M/G$, which shows that the quotient M/G is $\mathbb{P}^n$. Evidently, G fixes the points $M \cap \{z_{n+1} = 0\}$ and the image of this set under φ is H. In other words, $\varphi : M \to \mathbb{P}^n$ is a Galois covering of degree $n+2$, branched along the hypersurface H.

Let (M, G) be a pair with $M/G \simeq \mathbb{P}^n$. The corresponding projection map $\varphi : M \to \mathbb{P}^n$ induces an orbifold structure $(\mathbb{P}^n, \beta_\varphi)$ on $\mathbb{P}^n$, where $\beta_\varphi : \mathbb{P}^n \to \mathbb{N}$ is the map sending $p \in \mathbb{P}^n$ to the order of the stabilizer $G_q \subset G$, where q is a point in $\varphi^{-1}(p)$. In the example given above, the induced orbifold is $(\mathbb{P}^n, \beta_\varphi)$, where

$$\beta_\varphi(p) := \begin{cases} n+2 & p \in S \\ 1 & p \notin S \end{cases}$$

The *locus* of an orbifold $(\mathbb{P}^n, \beta)$ is defined to be the hypersurface $\mathrm{supp}(\beta - 1)$. In our case, the locus $(\mathbb{P}^n, \beta_\varphi)$ is precisely the hypersurface S.

We call an orbifold uniformized by a Calabi-Yau a *Calabi-Yau orbifold*. In order to study the pairs (M, G) with $M/G \simeq \mathbb{P}^n$, one can alternatively study the Calabi-Yau orbifolds $(\mathbb{P}^n, \beta)$. This is the approach taken in this project. Let $\mathcal{O} := (\mathbb{P}^n, \beta)$ be a Calabi-Yau orbifold and let X be a uniformization of $\mathcal{O}$. Since X is a Calabi-Yau manifold, the top Chern number $c_n(X)$ must vanish. This imposes a severe restriction on the set of admissible Calabi-Yau orbifolds $(\mathbb{P}^n, \beta)$. For example, the degree of the locus of $(\mathbb{P}^n, \beta)$ cannot exceed $2(n+1)$.

In dimension one, the classification of Calabi-Yau orbifolds is classical. In dimension two, there are further restrictions imposed by the fact that the euler number of a K3 surface is 24. This makes it possible to classify all K3 orbifolds with a locus of degree ≤ 5 (see [1]). There are no K3 orbifolds with a locus of degree > 6.

For general n, Calabi-Yau orbifolds on $\mathbb{P}^n$ are not well studied, but we believe that they can be effectively classified. In this work, we classify Calabi-Yau orbifolds on $\mathbb{P}^n$ under the assumption that the uniformizing group is abelian.

Some abelian Calabi-Yau orbifolds $\mathcal{O}$ appearing in the classification admits an action by a finite group G, and the quotient $\mathcal{O}/G$ is another Calabi-Yau orbifold. The uniformizing group of $\mathcal{O}/G$ is the extension of the uniformizing group of $\mathcal{O}$, and is usually non-abelian. This gives a way to construct some non-abelian Calabi-Yau orbifolds. In particular, we give a Calabi-Yau orbifold whose locus is given by the equation

$$xyzt(\sqrt{x} + \sqrt{y} + \sqrt{z} + \sqrt{t}) = 0.$$

This orbifold is invariant under the action of the symmetric group on four letters. The quotient is an orbifold with a singular base space.

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Free Group Actions on Products of Spheres

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Given a finite group G, we know that it can act freely and smoothly on a product of spheres, the challenging problem is to find the minimum integer k such that G acts freely and smoothly on a product of k spheres. I will discuss some recent progress in constructing such actions using representation theory and periodicity methods.

Classes of mixed abelian groups

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All groups are mixed abelian groups with finite torsion-free rank; that is, $r_0(G) := rank[G/T(G)] < \infty$. Equivalently, there exists a (finite rank) free subgroup $F \cong Z^n \leq G$ such that G/F is a torsion group.

We consider three classes of these groups, listed from the largest to the smallest:

$\mathcal{S}$: the class of self-small (**ss**) groups G : those for which $Hom(G, \oplus_{i \in I} G_i) \cong \oplus_{i \in I} Hom(G, G_i)$ for any index set I. Put another way, these are the groups for which any homomorphism from G into $\oplus_{i \in I} G_i$ has finite support. This is an extensive class of groups, containing all groups with countable endomorphism ring. Self-small mixed groups were first investigated by Arnold and Murley in 1975.

$\mathcal{D}$: the class of quotient divisible (**qd**) groups: those groups G such that G is an extension of some finite rank free subgroup by a divisible torsion group. Those qd groups G with G is torsion-free have been studied since the 1960's with initial investigations by Beaumont and Pierce. But we need not assume a qd group G is torsion-free as long as we add the condition that G contain no torsion divisible subgroup. Mixed qd groups were first considered by Fomin and Wickless in 1998 Probably the first major result on qd groups is that there is a duality between the category of torsion-free finite rank (**tffr**) groups and quasi-homomorphisms and the category of mixed qd groups and quasi-homomorphisms. A **quasi-homomorphism** from an abelian group A to an abelian group B is an element of $Q \otimes Hom(A, B)$. Thus, qd groups look like tffr groups at the "quasi-level". But under finer considerations qd and tffr groups can be quite different.

$\mathcal{G}$: the class of groups G which can be embedded into the direct product of their p-torsion subgroups: $\bigoplus_p T_p(G) \leq G \leq \prod T_p(G)$ satisfying the **projection condition**. The projection condition says that that there is a finite rank free subgroup $F \leq G$ that projects to a set of generators for almost all $T_p(G)$. For the finitely many exceptions, $T_p(G)$ is required to be finite. This class is the nicest of the three and has been studied by a number of authors over the last 15 years.

Let E be the endomorphism ring of an abelian group G. We consider our mixed group classes with respect to three long-standing abelian group/module type questions.

1. What are the groups G in some class such that the ring E has certain ring theoretic properties or such that the module ${}_EG$ has certain module theoretic properties?

2. (Kaplansky's question) If E' is the endomorphism ring of G' (with G, G' in some class of abelian groups) and $E \cong E'$ as rings, when is it true that $G \cong G'$?

3. What kind of uniqueness properties are there for direct sum decompositions? For a given class, Is there any sort of Krull-Schmidt theorem?

The talk will be at a basic level. I'll define everything that needs to be defined.

Coprime Comodules and Coalgebras

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Let A be an associative ring. An *A-coring* is an (A, A)-bimodule $\mathcal{C}$ with (A, A)-bilinear maps

$$\underline{\Delta} : \mathcal{C} \to \mathcal{C} \otimes_A \mathcal{C} \quad \text{and} \quad \underline{\varepsilon} : \mathcal{C} \to A,$$

called *coproduct* and *counit*, with the properties

$$(I_{\mathcal{C}} \otimes \underline{\Delta}) \circ \underline{\Delta} = (\underline{\Delta} \otimes I_{\mathcal{C}}) \circ \underline{\Delta} \text{ and } (I_{\mathcal{C}} \otimes \underline{\varepsilon}) \circ \underline{\Delta} = I_{\mathcal{C}} = (\underline{\varepsilon} \otimes I_{\mathcal{C}}) \circ \underline{\Delta}.$$

Associated to any coring are the *dual rings*, ${}^*\mathcal{C} = {}_A\mathrm{Hom}(\mathcal{C}, A)$ with unit $\underline{\varepsilon}$ and the product

$$f *^l g \; : \; \mathcal{C} \xrightarrow{\underline{\Delta}} \mathcal{C} \otimes_A \mathcal{C} \xrightarrow{I_{\mathcal{C}} \otimes g} \mathcal{C} \xrightarrow{f} A,$$

and $\mathcal{C}^* = \mathrm{Hom}_A(\mathcal{C}, A)$ with symmetric multiplication.

Right $\mathcal{C}$-comodules are defined as right A-modules M with an A-linear map

$$\varrho^M : M \to M \otimes_A \mathcal{C}$$

satisfying coassociativity and counital conditions. *Comodule morphisms* are defined canonically and the category of right $\mathcal{C}$-comodules is denoted by $\mathbf{M}^{\mathcal{C}}$.

Any right $\mathcal{C}$-comodule M is a left ${}^*\mathcal{C}$-module by the action

$${}^*\mathcal{C} \otimes_R M \to M, \qquad f \otimes m \mapsto (I_M \otimes f) \circ \varrho^M(m).$$

Any morphism $h : M \to N$ in $\mathbf{M}^{\mathcal{C}}$ is a left ${}^*\mathcal{C}$-module morphism, thus

$$\mathrm{Hom}^{\mathcal{C}}(M, N) \subset {}_{^*\mathcal{C}}\mathrm{Hom}\,(M, N)$$

and there is a faithful functor

$$\mathbf{M}^{\mathcal{C}} \to \sigma[{}_{^*\mathcal{C}}\mathcal{C}] \subset {}_{^*\mathcal{C}}\mathbf{M}.$$

Notice that $\mathcal{C}$ is always a subgenerator in $\mathbf{M}^{\mathcal{C}}$ and the following are equivalent:

(a) For all $M, N \in \mathbf{M}^{\mathcal{C}}$, $\mathrm{Hom}^{\mathcal{C}}(M, N) = {}_{^*\mathcal{C}}\mathrm{Hom}(M, N)$;

(b) $\mathbf{M}^{\mathcal{C}}$ is a full subcategory of ${}_{^*\mathcal{C}}\mathbf{M}$;

(c) $\mathbf{M}^{\mathcal{C}} = \sigma[{}_{^*\mathcal{C}}\mathcal{C}]$;

(d) $\mathcal{C}$ is locally projective as a left A-module.

Recall that *prime commutative rings* can be characterized by various properties. In fact, for a commutative ring R, the following are equivalent:

(a) *For any* $a, b \in R$, $ab = 0$ *implies* $a = 0$ *or* $b = 0$*;*

(b) *for any ideals* $I, J \subset R$, $IJ = 0$ *implies* $I = 0$ *or* $J = 0$*;*

(c) *for every ideal* $I \subset R$, ${}_R I$ *is faithful;*

(d) *for every ideal* $I \subset R$, R *is* I*-cogenerated;*

(e) *for every ideal* $I \subset R$, $R \in \sigma[I]$.

Some of these properties may be formulated for non-commutative rings A and also for A-modules. Clearly they need not remain equivalent in the more general setting.

The importance of such *primeness conditions* for modules and rings rises the questions of the relevance of such conditions for comodules and corings. By the close relationship between $\mathcal{C}$-comodules and ${}^*\mathcal{C}$-modules, the notions can be transferred easily from modules to comodules (provided ${}_A\mathcal{C}$ is locally projective). It turns out that the primeness conditions are very restrictive for coalgebras whereas the dual versions of these conditions, they may be called *coprimeness conditions*, are covering wider classes of comodules and corings.

In the talk an outline of these ideas will be given including recent results by Indah Wijayanti.

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Generalized Burnside rings and group cohomology

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This is a presentation of a joint work with Robert Hartmann. Let G be a finite group and X be a G-set. Given a $\mathbb{Z}G$-module M, we define $H^*_X(G, M)$, the cohomology of G associated to X with coefficients in M, as the cohomology of cochain complex where n-cochains are the maps $f : G^n \times X \to M$, and the coboundry maps are given by

$$\begin{aligned}(\delta f)(g_0, \dots, g_n; x) =& g_0 \cdot f(g_1, \dots, g_n; x)\\ &- f(g_0 g_1, \dots, g_n; x)\\ &\vdots\\ &+ (-1)^n f(g_0, \dots, g_{n-1} g_n; x)\\ &+ (-1)^{n+1} f(g_0, \dots, g_{n-1}; g_n x).\end{aligned}$$

The cohomology group $H^n_X(G, M)$ can be described in terms of the usual group cohomology of subgroups of G. In particular, when X is the transitive G-set G/H, the cohomology group $H^n_{G/H}(G, M)$ is isomorphic to $H^n(H, M)$.

The Burnside ring $B(G)$ of a finite group G is defined as the Grothendieck ring of the isomorphism classes of G-sets where the addition is given by disjoint union and the multiplication is given by cartesian product. We generalize this definition as follows: A positioned G-set is a pair of the form (X, u), where X is a G-set and u is a class in $H^n_X(G, M)$. The set of isomorphism classes of positioned G-sets is a semi-ring with addition and multiplication defined by

$$[X, u] + [Y, v] = [X \amalg Y, u \oplus v]$$

$$[X, u] \cdot [Y, v] = [X \times Y, u \otimes v].$$

where $u \oplus v \in H^n_{X \amalg Y}(G, M)$ and $u \otimes v \in H^n_{X \times Y}(G, M)$ are defined in the appropriate way on the cochain level. The cohomological Burnside ring $B^n(G, M)$ of degree n of the group G with coefficients in M is defined as the Grothendieck ring of this semi-ring.

If A is a abelian group with trivial G-action then $B^1(G, A)$ is the same as the monomial Burnside ring over A defined by Dress [4] (see also [1], [2]). When $n = 0$, we can take M as the group G with conjugation action and identify $B^0(G, M)$ with the crossed Burnside ring $B^c(G)$ (see Bouc [3], Oda-Yoshida [5], [6]). We also give an interpretation of $B^2(G, M)$ in terms of twisted group rings when $M = k^\times$ is the unit group of a commutative ring.

Finally, we discuss the generalizations of the usual notions for Burnside rings to the cohomological Burnside rings, such as the ghost ring and the mark homomorphism.

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Recognition of the p-core in finite groups

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A *black box group* X is a finite group whose elements are encoded as 0-1 strings of uniform length and the group operations are perfomed by an oracle ('black box'). Given strings representing $g, h \in X$, the black box can compute the strings representing $g \cdot h$, g^{-1} and decide whether $g = h$. The natural task here is to find probabilistic algorithms which determines the isomorphism type, with some probability of error, of the group X with the given degree of certainty. There is a positive answer to this question in class of finite simple groups, namely, we can determine the isomorphism type of a finite simple group of Lie type, see [1], [2]. In this talk, I will present an algorithm which produces an element, if any exists, from the p-core, $O_p(X)$, of the black box group X where $X/O_p(X)$ is a finite simple group of Lie type of odd characteristic p. The algorithm involves repeated constructions of centralizers of involutions and properties of conjugacy classes in these groups. The algorithm here is tested more than a hundred thousand times in the package 'Groups, Algorithms and Programmaing' GAP for various extensions of all finite groups of Lie type.

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On Mackey Algebras: Clifford Theory and Group Gradings

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Clifford theory is a repertoire of reduction and extension techniques that is applicable in the context of a given normal subgroup N of a given finite group G. It relies largely on a theory of group graded algebras. Meanwhile, the theory of Mackey functors is a general scheme for dealing with conjugation, restriction and transfer in group representation theory and cohomology. We introduce a Clifford theory of Mackey functors. The key idea is to consider an algebra $\mu(G,N)$ which is group graded over the Mackey algebra $\mu(N)$ and which is also a truncated subalgebra of $\mu(G)$. As applications, we obtain some reduction results and some extension results for Mackey functors, and also a Mackey functor analogue of Green's indecomposibility criterion.

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5 Schedule of talks (short form)

A more detailed schedule is on p. 8.

	Wed 18th	Thurs 19th	Fri 20th	Sat 21st	Sun 22nd
9:00	J. Murre	A. Scholl	P. Smith	V. Drensky	R. Thomas
10:00	K. Nicholson	J. Lewis	R. Wisbauer	V. Mazurov	P. Pragacz
11:00	coffee				
11:30	T. Albu	V. Srinivas	W. Wickless	S. Thomas	A. Klyachko
12:30	lunch				
14:00	Z.-L. Dou	R. Hartmann	recreation	M. Kerr	farewell!
15:00	parallel sessions (see below)			|| sessions (below)	
				18:50: class: *t.v.* III	
	dinner				
20:30		evening class: *torsal varyeteler* I, II			

	Wed		Thurs		Sat	
	1	2	1	2	1	2
15:00	İ. İkeda	N. Çağman	V. Tolstykh	R. Alizade	G. Karaali	∗
15:30	İ. Cangül	G. Alptekin	A. Berkman	E. Mermut	Y. Ozan	
16:00	coffee					
16:30	M. Uludağ	Ç. Özcan	S. Azgın	Ö. Ünlü	H. Ayık	†
17:00	M. Tosun	N. Orhan	D. Pierce	L. Barker	G. Ayık	
17:30	short break					
17.40	V. Micale	F. Özbudak	Ts. Rashkova	A. Abdollahi	R. Şahin	‡
18.10	B. Khan	F. Koyuncu	K. Rangaswamy	A. Madanshekaf	M. Kanuni	
18.40	short break					
18.50	V. Levchuk	Ş. Yalçınkaya	A. Fomin	E. Yalçın		

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