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Abstracts

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Contents

On Fuzzy Relation and Fuzzy Quotient Groups

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The purpose of this paper is to define the fuzzy quotient group by using some special fuzzy relation which is defined in this study and to prove some basic properties.

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Prof. M.G. Ikeda's Work on Cryptography and Coding Theory

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The purpose of this talk is to explain Ikeda's work on Cryptography and Coding theory. We first explain M.G. Ikeda's contribution to the Generalized Bent Functions in Cryptography. Then we shall introduce the system of weight equations for a binary (n,m)-code C with respect to its ordered basis $\Omega = \{u_1, \ldots, u_m\}$ and the set of characteristics of Ω. Finally we will explain their relations to the Bent functions and some combinatorial problems concerning binary vector spaces.

Krull dimension, dual Krull dimension, and quotient finite dimensionality

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A right R-module M is said to be *quotient finite dimensional* (QFD) if M/N has finite Goldie dimension for each submodule N of M. Roughly speaking the *Krull dimension* (resp. the *dual Krull dimension*) of M is a measure of how close M is to being Artinian (resp. Noetherian). Any module having Krull dimension or dual Krull dimension is QFD, but, in general, not conversely. We extend a series of results about QFD modules to upper continuous modular lattices by using Lemonnier's Lemma. Then, we present some results relating Krull dimension, dual Krull dimension, and the QFD property of upper continuous modular lattices. Applications are given to Grothendieck categories and module categories equipped with a torsion theory.

The results which will be presented have been obtained jointly with *Mihai Iosif* (Bucharest University, Romania) and *Mark L. Teply* (University of Wisconsin-Milwaukee, USA).

Tamsayılardan Latislere Bir Gezinti: GCD Matrisleri ve Uygulamaları

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$S = \{x_1, x_2, \ldots, x_n\}$ elemanları pozitif tamsayılar olmak üzere, x_i ve x_j pozitif tamsayılarının en büyük ortak bölenini ve en küçük ortak katını sırasıyla, (x_i, x_j) ve $[x_i, x_j]$ ile gösterelim. f bir aritmetik fonksiyon olmak üzere $ij-$yinci elemanı sırasıyla $f((x_i, x_j))$ ve $f([x_i, x_j])$ olan $n \times n$ tipinden $(S)_f$ ve $[S]_f$ matrislerine f fonksiyonuna göre S kümesi üzerinde sırasıyla GCD ve LCM matrisi denir [3, 4]. Bu matrisler üzerinde yapılan çalışmalarda, bu matrislerin determinantları ve terslerinin elemanları Euler'in toplam fonksiyonu ϕ ve Möbius fonksiyonu μ gibi bazı özel aritmetik fonksiyonlar cinsinden ifade edilmiş ve bu matrislerin bir çok genelleştirilmesi verilmiştir. Bu genelleştirilmelerin içinde en ilginci tamsayılar kümesinde bölünebilmenin bir kısmi sıralama bağıntısı olmasından hareketle, bu matrislerin latisler üzerine yapılan genelleştirilmesidir [1]. $S = \{x_1, x_2, \ldots, x_n\}$ kümesi yerine, bir $(P, \preccurlyeq)$ latisinin herhangi bir alt kümesinin alınması ve f aritmetik fonksiyonunun $(P, \preccurlyeq)$ latisi üzerinde tanımlanması ile GCD ve LCM matrisi sırasıyla meet ve join matrisine dönüşür [5, 6, 7]. Bu çalışmada GCD ve LCM matrisinin bu yöndeki genelleştirilmesi aktarılacak ve konu ile ilgili son gelişmeler tartışılacaktır [1, 2, 8, 9].

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The Relationship Between Special Groups and Dunwoody Parameters

NURULLAH ANKARALIOĞLU, İNCI GÜLTEKIN AND HÜSEYIN AYDIN

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It has been found in [1] and [2] that Cyclically presented group obtained by using the word w constituted as connected with Dunwoody parameters (1,0,1,2) is Fibonacci group. In this paper, it is shown that according to the results obtained from computer program, (a, b, c, r) parameters which satisfy Dunwoody conditions are

$$(m+1, m, k(4m^2+6m+2)+m+1, 4m^2+5m+2) \quad \text{and}$$
$$(m+1, m, k(4m^2+6m+2)+4m^2-m-3, k(4m^2+6m+2)-5m-3)$$

and torus knots corresponding to these parameters are

$$t(3m+2, k(3m+2)+3) \quad \text{and} \quad t(3m+2, k(3m+2)+3m-1)$$

respectively. Also, we will investigate special groups associated with cyclically presented groups obtained by using the word w constituted as connected with these parameters.

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A contribution to the Characterization of Locally Finite Minimal Non FC-Groups

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Let G be a group. If every element of G has a finite conjugacy class in G, then G is called an FC-group. If G is not an FC-group but every proper subgroup of it is an FC-group, then G is called a minimal non FC-group. Thus a minimal non FC-group is a generalization of a group of Miller–Moreno type, where a group is of Miller–Moreno type if it has infinite commutator subgroup but every proper subgroup of it has finite commutator subgroup.

The structure of locally finite groups of Miller–Moreno type were completely described by Belyaev and Sesekin. These groups are finite extensions of divisible abelian q-groups of finite rank. Thus they are Chernikov groups.

Let G be a locally finite minimal non FC-group. Belyaev has shown that if $G' < G$, then G is of Miller–Moreno type and if $G' = G$, then either $G/Z(G)$ is simple or G is p-group, for some prime p. Kuzucuoğlu and Phillips have shown that such a group must be a p-group.

Recently Belyaev has shown that if there is a perfect locally finite minimal non FC-group that is also a p-group, then it must have a nontrivial representation as a finitary permutation group.

In the present work it is shown that a perfect locally finite minimal non FC-group that is also a p-group cannot exist. Thus this result complements the earlier results in this direction, and so it follows that a locally finite minimal non-FC-group is a finite extension of a divisible abelian q-group of finite rank, *i.e.* it is a Chernikov group.

On Semigroup Presentations

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The aim of this talk is to give a survey of the works on finite presentability of semigroup constructions, minimal semigroup presentations, semigroup presentations of groups and monoids, and a Method which tests if a semigroup presentation defines a group.

We give the necessary and sufficient conditions for finite presentability of Rees Matrix Semigroups, strong semilattices of semigroups, etc., and necessary conditions for finite presentability of extension of a semigroup by a congruence. We also talk about the second integral homologies of semigroups which is known as Schur Multiplier in Group Theory. There also exists a connection between the second integral homologies and semigroup presentations. The second integral homologies of some semigroup families have been computed, and applied to computational semigroup theory.

There are some methods for obtaining semigroup presentation for groups (monoids) from a given group (monoid) presentation without changing the numbers of generators and relations. We talk about these methods and illustrate these Method some special classes of groups (monoids).

We find a connection between the Adian graphs and semigroups presentations. By using the Adian graphs we check if a semigroup presentation defines a group.

Actions on Stone spaces and co-Galois groups

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Let E/F be a Galois extension with Galois group $\Gamma = \mathrm{Gal}(E/F)$, and let X denote an arbitrary Stone space with $\mathcal{A}(X)$ its group of all continuous automorphisms. Considering the trivial action of the profinite group Γ on the discrete group $\mathcal{A}(X)$, the pointed set $H^1(\Gamma, \mathcal{A}(X))$ classifies up to an F-isomorphism the commutative F-algebras L which are regular in the sense of von Neumann such that for any maximal ideal $\mathbf{m}$, the residue field $L/\mathbf{m}$ is F-embeddable into E, and $\mathrm{Hom}_F(L, E)$, with the natural Stone topology, is homeomorphic to X.

For any such F-algebra L, Γ acts continuously on the E-algebra

$$\tilde{L} = L \otimes_F E \cong C(X, E),$$

and $L = \tilde{L}^{\Gamma}$. The torsion part of the factor group $\tilde{L}^*/L^*$, called the *co-Galois group* $\mathrm{Cog}(\tilde{L}/L)$ of $\tilde{L}/L$, is canonically isomorphic with $Z^1(\Gamma, C(X, \mu_E))$, where L^*, resp. μ_E denotes the multiplicative group of the invertible elements of L, resp. of all roots of unity contained in E.

The aim of this talk is to discuss the main properties of the groups as defined above in the frame of the so called *abstract co-Galois theory*. Some references related to the topic above are mentioned bellow.

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Approximation principle for Frobenius algebraic sets in Witt vectors

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We will discusss the possibility of an approximation principle in Witt vectors for sets defined by polynomials involving the Frobenius automorphism, in relation with a Nullstellensatz.

Discriminating and square-like groups

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The notions of discriminating and square-like groups were introduced and studied in a series of papers by Baumslag–Myasnikov–Remeslennikov and Fine–Gaglione–Myasnikov–Spellman [1, 2, 3, 4]. A group G is called *discriminating* [1] if every group separated by G is discriminated by G. Here G is said to *separate* (*discriminate*) a group H if for any non-identity element (finite set of non-identity elements) of H there is a homomorphism from H to G which does not kill the element (each element of the set). A group is discriminating iff it discriminates its direct square [1, 2]. A group is called *square-like* [3] if it is universally equivalent to its direct square. Any discriminating group is square-like [2].

Theorem 1. *Any square-like group is elementarily equivalent to a countable discriminating group.*

This answers a question in [3, 4]; the paper [4] is devoted to a proof of the result in the special case of abelian groups. The class of square-like groups is known to be axiomatizable [3]; we provide a nice explicite axiomatization.

Theorem 2. *A group is square-like if and only if it satisfies all first order sentences of the form* $\forall\bar{x}(\phi \to \bigvee_i \psi_i) \to \bigvee_i \forall\bar{x}(\phi \to \psi_i)$, *where* ϕ *is a conjunction of atomic formulas, and* ψ_i *are atomic formulas.*

Theorem 3. *The theory of the class of discriminating groups is computably enumerable but undecidable.*

We suggest a method of constructing discriminating groups and use it to construct in various group varieties plenty of discriminating nonabelian groups that do not embed their squares. (Note that it was mentioned in [2] that examples of discriminating nonabelian groups that do not embed their squares were not known). We construct square-like, non-discriminating nilpotent p-groups of arbitrary nilpotency class; all previously known square-like, non-discriminating groups were abelian [2].

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The contribution of Masatoshi Gündüz Ikeda in number theory

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The contribution of M. G. Ikeda in number theory is concentrated on the inverse Galois problem and the absolute Galois group $G_{\mathbf{Q}}$ of the rational number field $\mathbf{Q}$. In 1960, he proved that if a finite embedding problem with abelian kernel has a weak solution, then it has a strong solution over $\mathbf{Q}$. He generalized the Grunwald existence theorem in 1963 and the Hasse–Arf theorem in 1972. In 1975–77, he proved the famous conjecture of Neukirch that the continuous automorphisms of $G_{\mathbf{Q}}$ are inner automorphisms.

In this talk, we shall concentrate on his work concerning the embedding problem with abelian kernel and the completeness of the absolute Galois group of the rational number field.

A Note on the Geometric Mean-Reciprocal GCD and LCM Matrices

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In this paper we define an $n \times n$ matrix $[R] = (r_{ij})$, where

$$r_{ij} = \frac{\sqrt{x_i x_j}}{(x_i, x_j)},$$

and call it the Geometric Mean-Reciprocal GCD matrix on $S = \{x_1, x_2, \ldots, x_n\}$; and we define the $n \times n$ matrix $(S^{**}) = \left(s_{ij}^{**}\right)$, where

$$s_{ij}^{**} = \frac{\sqrt{x_i x_j}}{(x_i, x_j)^{**}},$$

calling it the Geometric Mean-Reciprocal GCUD matrix on gcud closed set of $S = \{x_1, x_2, \ldots, x_n\}$. In the second section we calculate the determinant, and the inverse of the Geometric Mean-Reciprocal GCD matrix on FC set S by the arithmetical function g and μ, Möbius function. In the third section we calculate the determinant, and the inverse of the Geometric Mean-Reciprocal GCUD matrix on gcud set S by the arithmetical function g and μ^*.

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The p-Cockcroft Property of the Semi-Direct Products of Monoids

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The semi-direct product of arbitrary two monoids and a presentation for this product have received considerable attention, see for instance [1], [4], [6] and [7]. In [7], Wang defined a trivialiser set of the Squier complex associated with this presentation. In this talk, as a main result, we discuss necessary and sufficient conditions for the standard presentation of the semi-direct product of any two monoids to be p-Cockcroft for any prime p or 0.

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On finite groups admitting a fixed point free automorphism of order pqr

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Let G be a finite group and A be a group of operators of G and $C_G(A) = 1$. Turull obtained that Fitting height of G is at most $l(A)$ where $l(A)$ denotes the length of the longest chain of subgroups of A, with certain exceptions for A, under the much used assumption that $(|G|, |A|) = 1$. We expect a similar bound for the Fitting height of G when we replace the assomption $(|G|, |A|) = 1$ by the assumption that A is nilpotent. This talk is about the following partial answer as a result of a joint work together with Prof. Güloğlu in this frame.

A finite group G admitting a fixed point free automorphism of order pqr for pairwise distinct primes p, q and r has fitting height at most 3.

On the other hand, by the use of the essential part of the proof, we are able to show the following result which answers a length type problem without coprimeness condition:

Let $H = G\langle\alpha\rangle$ where G is a soluble normal subgroup of H and $|\alpha| = p$, a prime. Assume that p^2 does not divide the order of any element $x \in H - G$. If $C_{A/B}(x)$ is nilpotent for any $\langle x\rangle$-invariant section A/B of G and for any $x \in H - G$ of order p, then the fitting height of G is at most 3.

Some applications of the theory of commutative monoids to the study of direct sum decompositions of modules

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For any ring R, let $V(R)$ denote the set of all isomorphism classes $\langle A_R\rangle$ of finitely generated projective right R-modules A_R. The set $V(R)$ with the addition induced by direct sum turns out to be a commutative monoid with order-unit $\langle R_R\rangle$. The monoid $V(R)$ is the pull-back of $V(R/J(R))$ and $K_0(R))$ over $K_0(R/J(R))$, where $J(R)$ and K_0 denote the Jacobson radical and the Grothendieck group respectively. For a module M_S over any ring S, the monoid with order-unit $(V(R), \langle R_R \rangle)$, where $R = \mathrm{End}(M_S)$ is the endomorphism ring of M_S, describes the direct sum decompositions of M_S. We shall prove how some notions of commutative monoid theory, like prime ideals, localizations, divisor homomorphisms, and so on, applied to the monoid $V(R)$, find natural interpretations and applications in the study of the ring structure of R.

Description of Modal Logics Inheriting Admissible Rules for $K4$

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We give the notion of co-cover property for logics with fmp extending $K4$ and necessary and sufficient condition for any modal logic with fmp to inherit all inference rules admissible in $K4$.

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Simple modules over small von Neumann regular rings

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Over a von Neumann regular ring (or simply a regular ring) with bounded index of nilpotency all simple modules are injective, that is, they are V-rings. In general this is not true, but an internal characterization of regular V-rings is far from being known.

Assume R is a regular ring with infinite index of nilpotency that, in addition, is $\aleph_0$-injective or $\aleph_0$-continuous. We show that if R is a V-ring then it must have a simple module whose dimension over its endomorphism ring is at least $2^{2^{\aleph_0}}$. Hence *small*, $\aleph_0$-injective or $\aleph_0$-continuous, regular rings with infinite index of nilpotency are not V-rings. The main step in the proof of the result relies on a counting argument based on a Lemma by Tarski, the method is patterned in ideas first used by Osofsky.

In a much elementary way, we show that if R is a simple regular ring that is a countably generated algebra over a commutative field K then none of its simple modules can be injective unless it is semisimple artinian. This leads us to study simple modules over countable dimensional simple ultramatricial K-algebras. We show that in some of these examples all division rings that are finite dimensional over K can be realized as endomorphism rings of suitable simple modules. We also describe the injective hulls of some simple modules.

Over this class of rings $\mathrm{Ext}^1_R(V_1, V_2) \neq 0$ for any pair of simple modules, this implies that modules of socle-height 2 are of wild representation type.

Discrete and quasi-discrete modules

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The aim of this talk is to show that discrete and quasi-discrete modules M can be characterized in terms of the lifting of homomorphisms from M to M/N, for certain submodules N of M.

Let n be a positive integer. We consider the following two conditions for a module M:

(S_n) For every submodule K of M such that $K = K_1 \cap \cdots \cap K_n$, where every K_i is a coclosed submodule in M and $M = K_i + \bigcap_{i \neq j} K_j$ $(1 \leq i \leq n)$, every homomorphism $\varphi : M \longrightarrow M/K$ can be lifted to a homomorphism $\theta : M \longrightarrow M$.

(T_n) For every submodule K of M such that $K = K_1 \cap \cdots \cap K_n$, where every K_i is a submodule of M with M/K_i isomorphic to a coclosed submodule of M and $M = K_i + \bigcap_{i \neq j} K_j$ $(1 \leq i \leq n)$, every homomorphism $\varphi : M \longrightarrow M/K$ can be lifted to a homomorphism $\theta : M \longrightarrow M$.

The main results of our work are:

Theorem 1. *The following are equivalent for an amply supplemented module M.*

(i) M is quasi-discrete.

(ii) M satisfies (S_n) for every positive integer n.

(iii) M satisfies (S_2).

Theorem 2. *For any module M consider the following statements.*

(i) M is discrete.

(ii) M satisfies (T_n) for every positive integer n.

(iii) M satisfies (T_2).

(iv) M satisfies (T_1).

Then (i) $\Longrightarrow$ (ii) $\Longrightarrow$ (iii) $\Longrightarrow$ (iv) holds. If M is lifting, then (iv) $\Longrightarrow$ (i).

On the Order-k Generalized Lucas Numbers

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In this paper we give a new generalization of the Lucas numbers in matrix representation. Also we present a relation between the generalized order-k Lucas sequences and Fibonacci sequences.

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Ikeda's Contribution to the Theory of Frobenius Algebras

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As a natural generalization of group algebras of finite groups, Frobenius algebras have a prominent place in the theory of algebras. The theory has been founded by Nakayama, Ikeda, Shoda, Nagao and others. In this short survey we shall try to outline the most striking points in Ikeda's papers on Frobenius algebras. A special emphasis will be given to the concepts appeared in the literature currently, originated from these papers.

Isomorphisms of the unitriangular groups and associated Lie rings for the exceptional dimensions

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For arbitrary chain Γ and associative ring K with identity 1_K the ring $NT(\Gamma, K)$ is generated by elements xe_{ij} $(x \in K,\ i, j \in \Gamma, i > j)$ with the usual rules of the addition and multiplication of elementary matrices; if $|\Gamma| = n < \infty$, then we write $NT(n, K)$. Let $R = NT(\Gamma, K)$. The adjoint group of the ring R is isomorphic to the unitriangular group $UT(\Gamma, K)$. Structural connections between the adjoint group $G(R)$ and associated Lie ring $\Lambda(R)$ of R are investigated in [4], see also [5].

Standard automorphisms and isomorphisms of the rings R, $\Lambda(R)$ and the adjoint group $G(R)$ were distinguished in $[2-4]$, see also [1]. Let $R' = NT(\Omega, S)$ for a chain Ω and an associative ring S with identity. By [4] and [2], if either $2 < |\Gamma| < \infty$ or K is a ring with no zero-divisors, then every isomorphism between rings R and R' is standard; the same is true for their adjoint groups and associated Lie rings at $|\Gamma| > 4$. It was shown in [3], for $|\Gamma| \leq 4$ there exist non-standard automorphisms of $G(R)$ and $\Lambda(R)$ even if the ring K is commutative. The aim of this study is to investigate isomorphisms in the exceptional cases.

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Locally finite groups of finite rank

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The following result will be discussed:

Let G be an infinite locally finite simple group admitting an involutory automorphism ϕ such that $C_G(\phi)$ is of finite rank. Then G is isomorphic to $PSL(2,K)$ for an infinite locally finite field K of odd characteristic and ϕ is induced by conjugation by an element of $PGL(2,K)$.

This is joint work with Shumyatsky.

Some Questions in Chevalley Group's Theory and in K-theory

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Let $UG(K)$ be the unipotent subgroup $\langle X_r \mid r \in G^+ \rangle$ of a Chevalley group $G(K)$ of a normal Lie type $G = \Phi$ or twisted type ${}^m\Phi$ over a field K and $U = UG(K)$.

Theorem 1. *Every pair unipotent intersection* $U \cap U^g$ $(g \in G(K))$ *is conjugated in* $G(K)$ *to a normal subgroup of* U *if and only if either the Lie rank of* $G(K)$ *is equal to 1 or* $G = A_2$ *or char* $K = 2$ *and* $G = B_2$ *or char* $K = 3$ *and* $G = G_2$.

The question about pair 2-Sylow intersections of finite groups is raised in [1, question 5.14]; the case of independent Sylow 2-subgroups had been studded by M. Suzuki (1964). The proof of Theorem 1 uses a description of normal subgroups of the group $UG(K)$ which is received mutually with a description of ideals of associated ring, see [2] – [5]. We also investigate some combinatorial questions in K-theory and P. Neumann's hypothesis [1, question 6.38].

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On the existence of finite Galois stable subgroups of GL_n

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Let E be a finite extension of a number field F with Galois group Γ, and let O_E and O_F be the maximal orders of E and F. Let O'_E be the intersection of valuation rings of all ramified prime ideals in the ring O_E, and let $O'_F = F \cap O'_E$. Denote $\phi_E(t) = [E(\zeta_t) : E]$ where ζ_t is a primitive t-root of 1. Let $F(G)$ be a field obtained via adjoining to F all matrix coefficients of all matrices $g \in G \subset GL_n(E)$.

Theorem 1. *1. For a given number field F and integers n and t, there is only a finite number of normal extensions E/F such that $E = F(G)$ and G is a finite abelian Γ-stable subgroup of $GL_n(O_E)$ of exponent t.*

2. For a given number field F and integers n and $d = [E\colon F]$, there is only a finite number of fields $E = F(G)$ for some finite Γ-stable subgroup G of $GL_n(O_E)$.

Theorem 2. *Let $d > 1, t > 1$ and $n \geq \phi_E(t)d$ be given integers, and let E/F be a given extension of degree d. Then there is an abelian Γ-stable subgroup $G \subset GL_n(E)$ of exponent t such that $E = F(G)$.*

Theorem 3. *Let $d > 1, t > 1$ be given rational integers, and let E/F be an unramified extension of degree d.*

1. *If $n \geq \phi_E(t)d$, there is a finite abelian Γ- stable subgroup $G \subset GL_n(O'_E)$ of exponent t such that $E = F(G)$.*
2. *If $n \geq \phi_E(t)dh$ and h is the exponent of the class group of F, there is a finite abelian Γ-stable subgroup $G \subset GL_n(O_E)$ of exponent t such that $E = F(G)$.*
3. *If $n \geq \phi_E(t)d$ and h is relatively prime to n, then G given in 1) is conjugate in $GL_n(F)$ to a subgroup of $GL_n(O_E)$.*

4. *If d is odd, then G given in 1) is conjugate in $GL_n(F)$ to a subgroup of $GL_n(O_E)$.*

In all cases above G can be constructed as a group generated by matrices $g^\gamma, \gamma \in \Gamma$ for some $g \in GL_n(E)$.

Theorem 4. *Let E/F be a given extension of degree d, and let $G \subset GL_n(E)$ be a finite abelian Γ-stable subgroup of exponent t such that $E = F(G)$ and n is the minimum possible. Then $n = d\phi_E(t)$ and G is irreducible under conjugation in $GL_n(F)$. Moreover, if G has the minimal possible order, then G is a group of type $(t, t, ..., t)$ and order t^m for some integer $m \leq d$.*

Theorem 5. *Let K/Q be a normal extension with Galois group Γ, and let $G \subset GL_n(O_K)$ be a finite Γ-stable subgroup. Then $G \subset GL_n(O_{K_{ab}})$ where K_{ab} is the maximal abelian over Q subfield of K.*

Infinite Frobenius groups and related topics

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Let G be a transitive permutation group on a (possibly, infinite) set Ω such that the stabilizer $H = G_\alpha$ of a point $\alpha \in \Omega$ is non-trivial but the stabilizer of every two distinct points is trivial. In particular, H is *malnormal* in G, that is H is a proper subgroup of G and $H \bigcap H^g = 1$ for every $g \notin H$. By famous result of G.Frobenius, if Ω is finite then H has a normal complement F in G consisting of trivial element and all elements in G which fix no points in Ω, and F is a regular subgroup, i.e. F is transitive and $F_\alpha = 1$. For infinite group G, the set $F = (G \setminus \bigcup_{\alpha \in \Omega} G_\alpha) \cup \{1\}$ is not necessarily a subgroup. In the case when F is a regular subgroup we call G a *Frobenius group* with *core* F and *complement* H.

The structure of finite Frobenius groups is well-studied. The present talk is devoted to infinite Frobenius groups whose complement contain a non-trivial element of small order.

Strongly $\oplus$-supplemented modules

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Definition. Let $U \leq M$ and $V \leq M$. If V is minimal with respect to $M = U + V$ then V is called a **supplement** of U in M. This is equivalent to $M = U + V$ and $U \cap V \ll V$. M is called **supplemented** if every submodule of M has a supplement in M. M is called $\oplus$–supplemented if every submodule of M has a supplement, that is a direct summand of M. Let $V \leq M$. V is called **lies above a direct summand** of M if there exist submodules M_1 and M_2 of M such that $M = M_1 \oplus M_2$, $M_1 \leq V$, $V \cap M_2 \ll M_2$. M is called **(D1)** module if every submodule of M lies above a direct summand of M. Let M be a supplemented module. If every supplement submodule of M is a direct summand of M then M is called a **strongly $\oplus$–supplemented module**.

Supplemented and π–projective modules are strongly $\oplus$–supplemented. (D1) modules are also strongly $\oplus$–supplemented. $\oplus$–supplemented modules can be found in [2] and [3].

Lemma 1. *Let M be strongly $\oplus$–supplemented module. Then every direct summand of M is strongly $\oplus$–supplemented.*

Corollary 2. *Strongly $\oplus$–supplemented modules are completely $\oplus$–supplemented.*

Theorem 3. *Let M_i are projective modules ($1 \leq i \leq n$). Then $\oplus_{i=1}^{n} M_i$ is strongly $\oplus$–supplemented if and only if every M_i is strongly $\oplus$–supplemented.*

Theorem 4. *Let M be a projective module, then the followings are equivalent:*

(i) M is semiperfect.

(ii) M is supplemented.

(iii) M is $\oplus$–supplemented.

(iv) M is strongly $\oplus$–supplemented.

Theorem 5. *A commutative ring R is semiperfect if and only if every π–projective cyclic R–module is strongly $\oplus$–supplemented.*

Theorem 6. *For a supplemented module M, the following statements are equivalent:*

(i) M is strongly $\oplus$–supplemented.

(ii) Every supplement submodule of M lies above a direct summand.

(iii) (a) Every nonzero supplement submodule of M contains a nonzero direct summand of M.

(b) Every supplement submodule of M contains a maximal direct summand of M.

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Generators of simple groups and their applications

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The report is devoted to a method of choosing generators of finite simple groups. This method is based on the discription of subgroups of Lie type groups, whose intersections with all root subgroups are nontrivial. Using this method, we give answers to some problems related to the search of generators of paticular order. Also, it is noted some applications of such generators in solving the inverse Galois problem and in theory of graphs.

Linear Unequal Error Protection Codes and Algebraic Curves

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We recall some items of Linear Unequal Error Protection Codes. We show that the concept of "generalized algebraic geometry codes" gives a natural framework for constructing linear unequal error protection codes. We also give some examples. This talk summarizes some results of a joint work with Henning Stichtenoth.

Semiregular modules with respect to a fully invariant submodule

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A left R–module M is called *($Rad(M)$–) semiregular* if for all $x \in M$ there exists a decomposition $M = A \oplus B$ such that $A \leq Rx$ is projective and $B \cap Rx \leq Rad(M)$. It is well known that the Jacobson radical $Rad(M)$ is a fully invariant submodule of M. In this work we consider any fully invariant submodule F of a module M instead of $Rad(M)$ and define F–semiregular modules. We investigate some equivalent conditions and some other certain fully invariant submodules.

Joint work with: M. Alkan (Hacettepe University)

A generalization of groups with all subgroups subnormal

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In this work we consider a generalization of groups with all subgroups subnormal. Let G be a group. If every proper subgroup of G has proper normal closure in G, that is, $H^G \neq G$ for every proper subgroup of G then what is the structure of G? We proved the following results:

If H^n is a hypercentral subgroup for all subgroups H where $n \in N$ then G can not perfect.

If there is a $d \in N$ such that $\Phi(x_1, x_2, \ldots, x_{2^d}) = 1$ where $x_1, x_2, \ldots, x_{2^d} \in H$ then G cannot perfect.

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On the model-theory of Lie-rings

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Most common algebraic theories have universal or, at worst, $\forall\exists$ axioms; therefore they have *existentially closed* models. The degree to which these models can be nicely characterized is one measure of the 'tameness' of the original theory. The theory of integral domains is quite tame in this regard, as its existentially closed models are just the algebraically closed fields; but group-theory is wilder.

Of the theory DF^m of fields with m commuting derivations, the existentially closed models can be given a *first-order* characterization in several ways (see [1] and references there): in model-theoretic terminology, DF^m has a *model-companion.* If commuting derivations on a field K span a space E of dimension m over K, then E is also a *Lie-ring* with respect to the bracket-operation. Additionally, E has a basis $\{\partial_i : i < m\}$, and K includes $\{t^j : j < m\}$, such that $\partial_i t^j = \delta_i^j$ in each case. The t^j are group-automorphisms of E; if b stands for the bracket-operation, then I call the structure

$$(E, \mathrm{b}, t^0, \dots, t^{m-1})$$

an *m-dimensional modular Lie-ring.* Since it is bi-interpretable, by existential formulas, with the differential field $(K, \partial_0, \dots, \partial_{m-1})$, the theory of m-dimensional modular Lie-rings has a model-companion. The pair (K, E) is symmetrical in the sense that, as rings, K embeds in $(\mathrm{End}(E), \circ)$, and E in $(\mathrm{End}(R), \mathrm{b})$, in a way that makes certain corresponding diagrams commutative.

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Generalized Permutation

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In this paper, we study the concept of a polygroup, which is a generalization of the concept of ordinary group. We give some applications and a few interesting examples. We also introduce the concept of generalized permutation and its directed graph and we obtain some results.

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Using formal concept analysis to find the congruence lattice of a finite algebra

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Properties of the congruence lattice of an algebra, such as distributivity, modularity and permutability, are important in the study of the given algebra and the variety generated by it. This lattice is a complete sublattice of the lattice of all equivalence relations.

In formal concept analysis it is shown that a complete sublattice of the conceptlattice of a context corresponds to a special subcontext. We show how this subcontext can be found for a given finite algebra.

The palindromic width of a free product of groups

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The results reproduced below obtained together with Valery Bardakov (Institute of Mathematics, Novosibirsk.)

Let G be a group and S a generating set of G with $S^{-1} = S$. By definition the *width* of G relatively to S is the least natural number k such that every element of G is a product of at most k elements of S, or ∞, otherwise. Suppose F is a free group. By the *primitive width* of F we mean the width of F with respect to its primitive elements. The problem of determination of the primitive width of a non-abelian finitely generated free group is open.

Proposition 1. *Let F_2 be a two-generator free group. Then the primitive width of F_2 is infinite.*

To prove the Proposition we reduce the problem to the determination of the palindromic width of F_2. We define the *palindromic width* of a free product

$$G = \prod_{i\in I}^{*} G_i \qquad (*)$$

of groups to be the width of G relatively to the set of all palindoromes. A element $g \in G$ is said to be a *palindrome associated with the free decomposition* $(*)$ if, being written as a reduced word in syllables from G_i, g reads the same forward and backward (like, for instance, a word $a_1b_2c_3b_2a_1$, where a_1, b_2, c_3 are non-identity elements from distinct free factors.) Palindromes of a free group F are palindromes associated with some free decomposition of F into a free product of cyclic groups.

Lemma 2. *Any primitive element of F_2 is a product of at most two palindromes.*

Therefore infiniteness of the palindromic width of F_2 implies infiniteness of the primitive width of F_2.

Theorem 3. *Let $G = \prod_{i\in I}^{*} G_i$ be a free product of non-identity groups. The palidromic width of G is infinite if and only if $|I| \geqslant 3$ or there is at least one free factor G_i having more than two elements.*

Corollary. *The palindromic width of any non-abelian free group is infinite.*